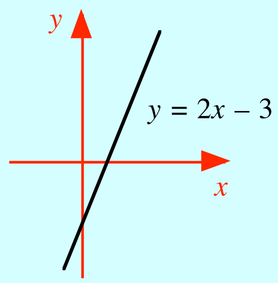
Introduction

We have already looked at the graphs of various functions :–

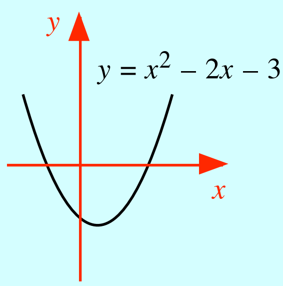
The **Linear** function

$f(x) = 2x - 3$



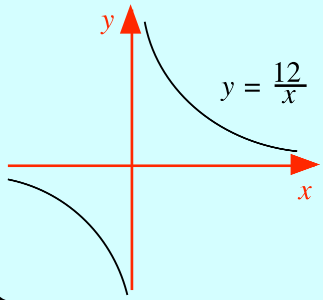
The **Quadratic** function

$f(x) = x^2 - 2x - 3$



The **Hyperbolic** function

$f(x) = \frac{12}{x}$



We will now begin to study the graphs of the trig functions, $y = \sin x°$, $y = \cos x°$ and $y = \tan x°$.

The Sine Function ($y = \sin x°$)

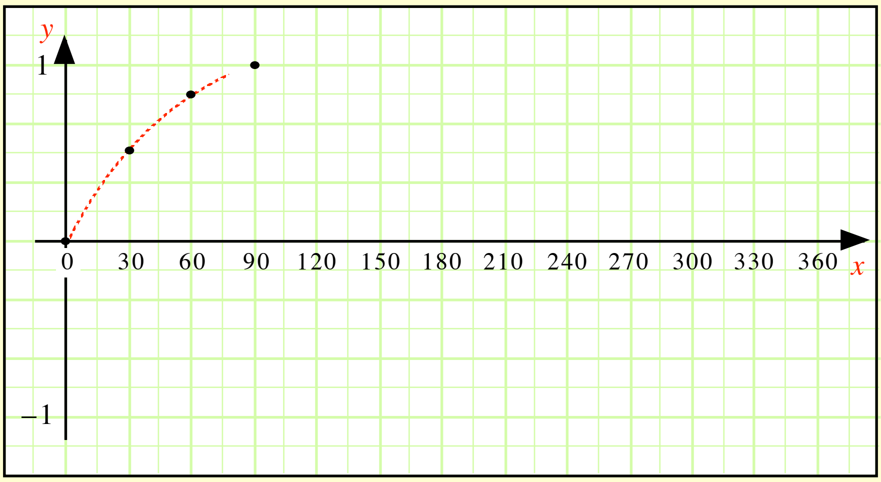
There are **computer** or **graphics packages** that could assist here.

You are going to look up the sine of various angles from 0° to 360° and plot them on a graph

(a) Copy this table and use your calculator to complete it (to 2 decimal places each time).

x	0	30	60	90	120	150	180	210	240	270	300	330	360
$\sin x°$	0	0·50	0·87	1·00	0·87	...	...	–0·50	...	...	...	...	...

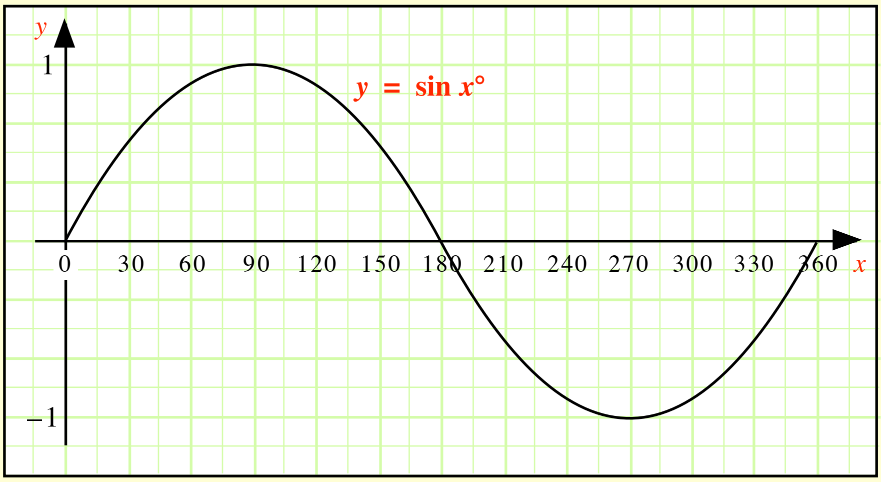
(b) Now take a sheet of A4 two millimetre graph paper and use it in the landscape position. Plot your 13 points from the table and join them up with a smooth curve.



Your graph should have ended up looking like this :-

$y = \sin x°$

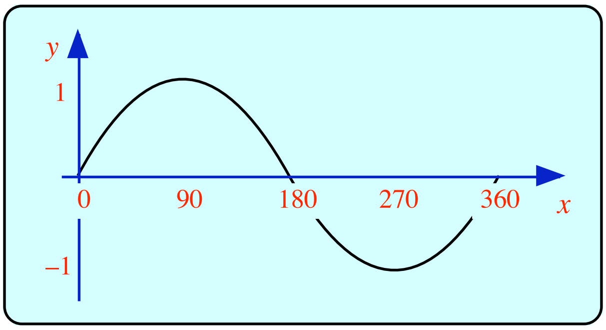
Study it carefully.



Exercise 7·1

1. (a) Practice sketching the graph **several** times on a smaller scale like this :-

   - note the smooth "wavy" shape.

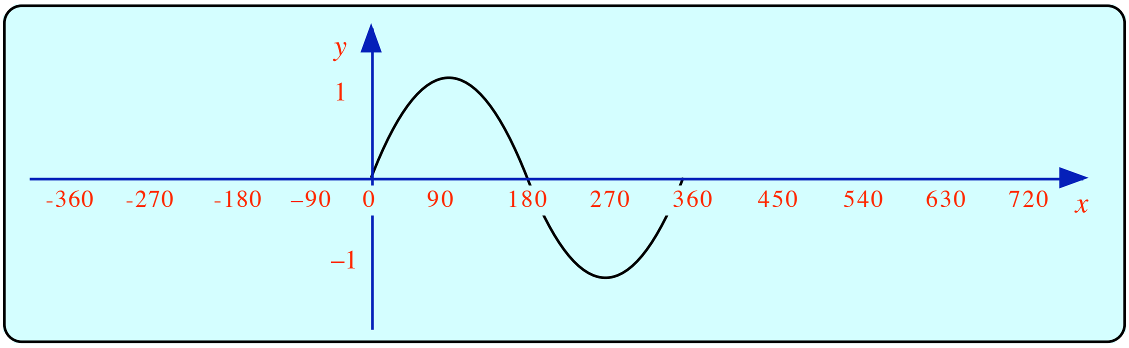


   (b) What is the **highest** value the graph attains ?

   (c) What is its **lowest** value ?

   (d) For what values of "x" does the curve cut the x-axis ?

2. (a) Make a new neat small sketch of $y = \sin x°$, but this time extend the x-axis to go from –360° to +720°. (*see below*).



   (b) Use your calculator to find sin 450°, sin 540°, sin 630° and sin 720°, plot these points on your diagram and sketch the next "bit" of the sine graph.

   (c) Repeat for sin (–90°), sin (–180°), sin (–270°) and sin (–360°) and draw this "bit" of the sine graph.

   (d) For the graph of $y = \sin x°$, state the **maximum** and **minimum** values . (*how high and low it goes*).

   (e) The "vertical distance" between the **maximum** and **minimum** values is called the **amplitude**. What is the **amplitude** of the sine graph ?

   (f) The "horizontal distance" between points on the graph where the pattern repeats itself is called the **period** of the graph. What is the **period** of the sine graph ?

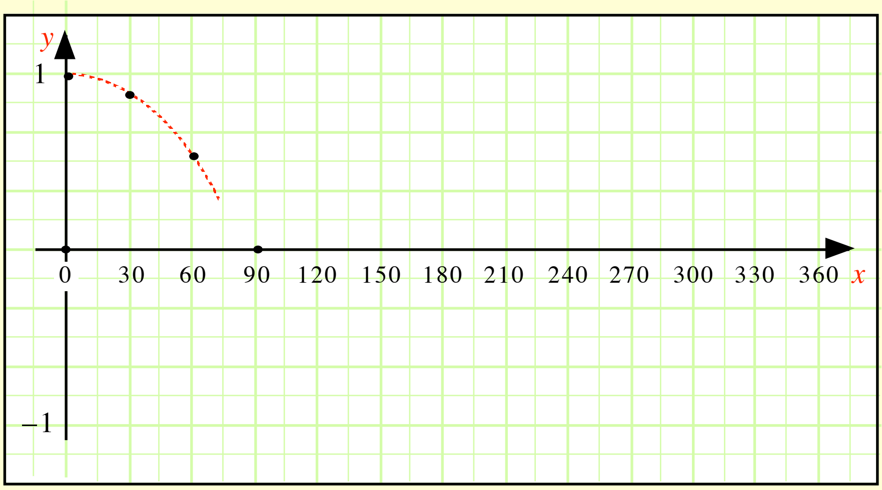
The Cosine Function ($y = \cos x°$)

You are going to look up the cosine of various angles from 0° to 360° and plot them on a graph.

(a) Copy this table and use your calculator to complete it (to 2 decimal places each time).

x	0	30	60	90	120	150	180	210	240	270	300	330	360
$\cos x°$	1	0·87	0·50	0	–0·50	...	...	...	...	...	...	...	...

(b) Now take a sheet of A4 two millimetre graph paper and use it in the landscape position. Plot your 13 points from the table and join them up with a smooth curve.



Show your finished smooth cosine graph to your teacher.

Exercise 7·2

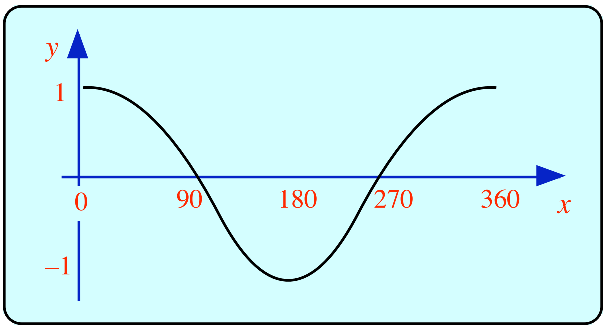
1. (a) Practice sketching the graph **several** times on a smaller scale like this :-

   - note again the smooth "wavy" shape.

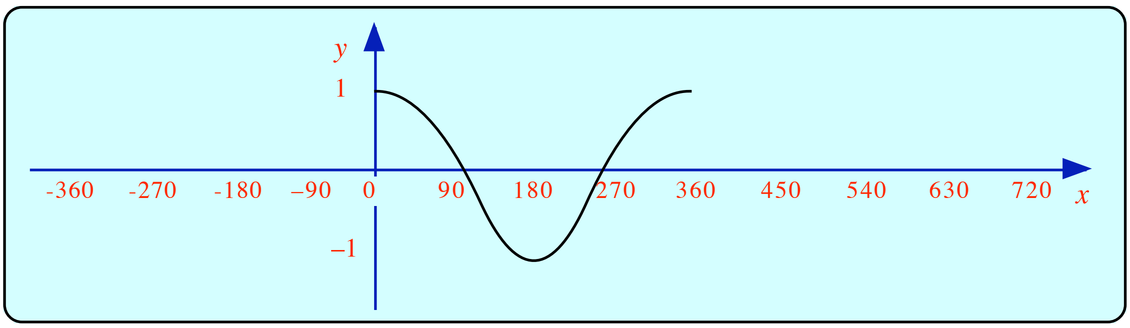
   (b) What is the **highest** value the graph attains ?

   (c) What is its **lowest** value ?

   (d) For what values of "x" does the curve cut the x-axis ?



2. (a) Make a new neat sketch of $y = \cos x°$, and extend the x-axis to go from –360° to +720°.



(b) Use your calculator to find cos 450°, cos 540°, cos 630° and cos 720°, plot these points on your diagram and sketch the next “bit” of the cosine graph.

(c) Repeat for cos (–90°), cos (–180°), cos (–270°) and cos (–360°) and draw this “bit” of the cosine graph.

(d) For the graph of **$y = \cos x°$**, state the **maximum** and **minimum** values. (*how high and low it goes*).

(e) The “vertical distance” between the **maximum** and **minimum** values is called the **amplitude**. What is the **amplitude** of the cosine graph ?

(f) The “horizontal distance” between points on the graph where the pattern repeats itself is called the **period** of the graph. What is the **period** of the cosine graph ?

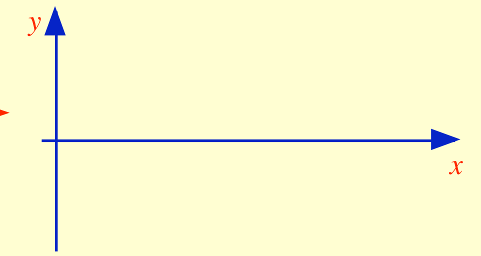
3. (a) Without looking at the last page, make a quick sketch of **$y = \sin x°$** and **$y = \cos x°$**, marking in the important values on both the x and the y-axes.

(b) Write a couple of sentences describing both graphs - in what ways are they similar and in what ways are they different ? (*shape, maximum/minimum values, amplitudes, periods*) ?

note :- when sketching any sine or cosine graph, it is easier if you do so in the following order :-

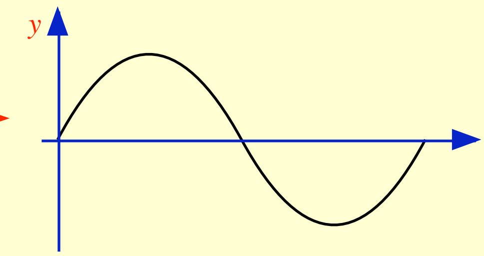
step 1

- draw the axes first.



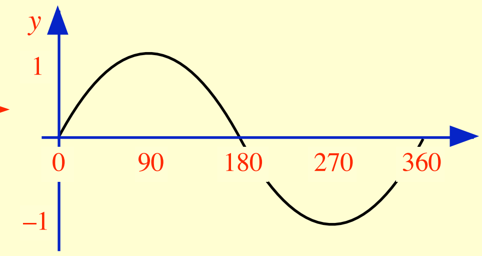
step 2

- **then** draw the sine, (or cosine), shaped graph.



step 3

- **finally** fill in the scales on the axes.



Try to remember to draw your trig graphs **in the above order**.

We will, through the course of the next 2 pages, draw graphs like :-

- **$y = 2\sin x°$**
- **$y = \sin 2x°$**
- **$y = 0{\cdot}3\sin 6x°$**
- **$y = \sin x° + 1$**
- **$y = -6\cos x°$**
- **$y = \cos 4x°$**
- **$y = -5\cos 12x°$**
- **$y = 5\cos x° - 5$**

but let us first of all look at the **tangent** graph.

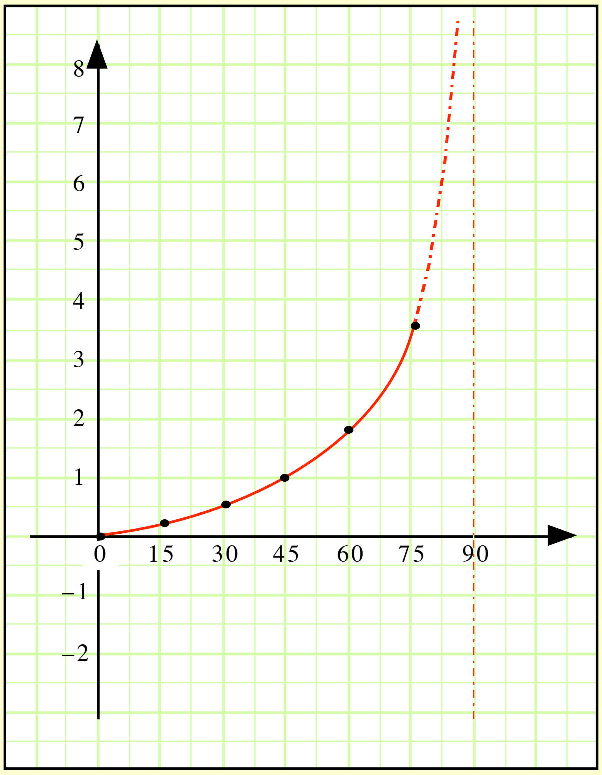
The Tangent Function (y = tan$x$°)

The tangent graph looks totally different from the sine and cosine graphs.

(a) Copy this table and use your calculator to complete it (to 2 decimal places each time).

x	0	15	30	45	60	75	89	90
tan$x$°	0	0·27	0·58	1·00	1·73	3·73	57·3	?

- At 90°, we say the tangent is **undefined** (it is too large a number to find - **infinity** !)



(b) Now take a sheet of A4 two millimetre graph paper (or half-cm paper) and use it in the portrait position.

Plot the 7 points from the table and join them up with a smooth curve.

- Note the shape - the slope rising very slowly at first, then accelerating towards infinity !

(c) Extend your table to show the x-values from 90° to 180°

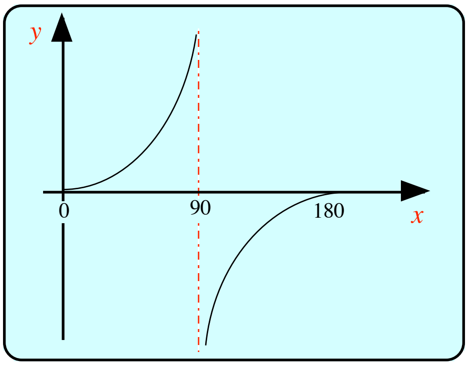
x	91	105	120	135	150	165	180
tan$x$°	-57·3	-3·73	-1·73	...	...	...	...

Extend your x-axis, plot these 6 points and join them to show the second half of the smooth tangent curve.

Show your finished smooth tangent graph to your teacher.

Exercise 7·3

1. (a) Practice sketching the graph **several** times on a smaller scale like this :-

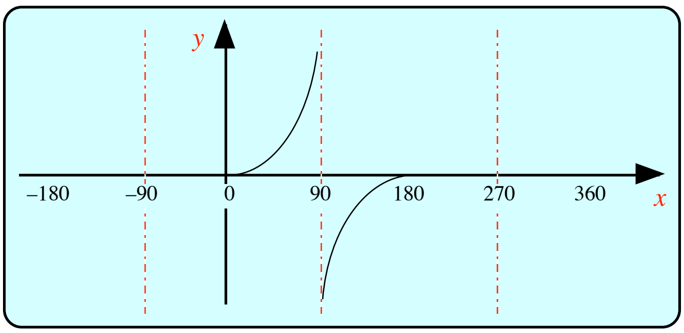


   (b) Is there a **highest** value the graph attains ?

   (c) Is there a **lowest** value ?

   (d) For what values of "x" does the curve cut the x-axis ?

2. (a) Make a new neat sketch of y = tan$x$° $\{0 \leq x \leq 180\}$ and extend the x-axis to go from –180° to +360°.



   (b) What is the **period** of the tangent function ?

   - The tangent function is not as important as the sine and cosine functions.
     It does not appear often in real life.

Other Sine and Cosine Functions ($y = a\sin x°$ and $y = a\cos x°$)

$y = 2\sin x°$

You are going to draw the graph of $y = 2\sin x°$ by looking up various values of x.

For example, when $x = 30$ => $\sin x° = 0{\cdot}5$ => $2\sin x° = 2 \times 0{\cdot}5 = 1$

(a) Copy this table and use your calculator to complete it (to 2 decimal places each time).

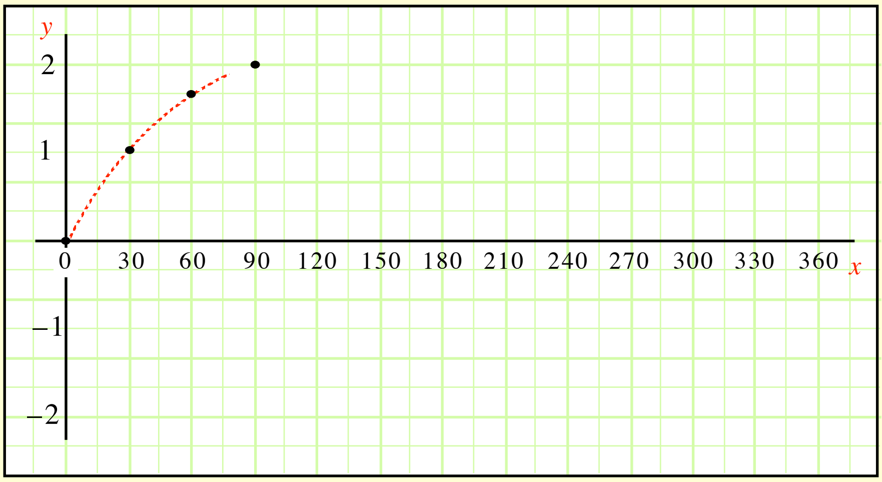
x	0	30	60	90	120	150	180	210	240	270	300	330	360
$2\sin x°$	0	1·00	1·73	2·00	1·73	...	...	–1·00	...	...	...	...	...

(b) Draw a set of axes on squared paper, plot the above 13 points and join them up with a smooth curve.

Show your graph of :-

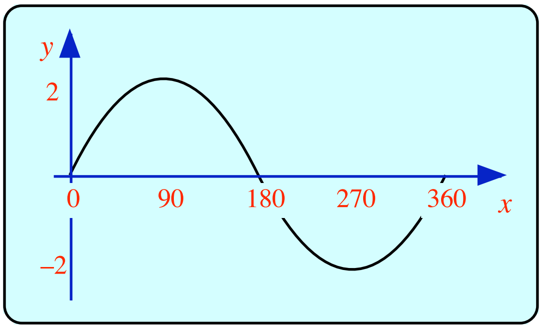
$y = 2\sin x°$

to your teacher.



Exercise 7·4

1. (a) This time, just sketch the graph $y = 2\sin x°$ on a smaller scale like this :-



(b) What is the **highest** value the graph attains ?

(c) What is its **lowest** value ?

(d) What is the **period** of $y = 2\sin x°$?

(e) For what values of "x" does the curve cut the x-axis ?

You should have noticed the following :-

- the graph is **identical** in shape to that of $y = \sin x°$.
- its **maximum** and **minimum** values are now +2 and –2, and its **amplitude** becomes 4.
- its period is still 360° – it is not altered by the $2\sin x°$.

You will find :- the maximum/minimum value of $y = 3\sin x°$ is +3 and –3. Its period is still 360°

the maximum/minimum value of $y = -5\sin x°$ is +5 and –5. Its period is still 360°

the maximum/minimum value of $y = 10\cos x°$ is +10 and –10. Its period is still 360°

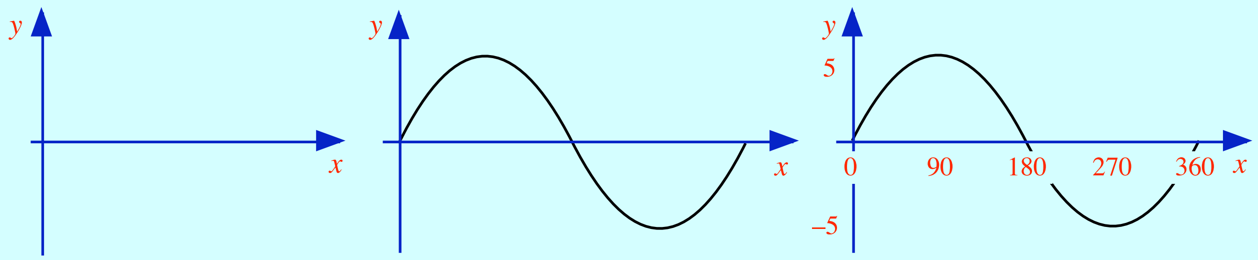
the maximum/minimum value of $y = \frac{1}{2}\sin x°$ is $+\frac{1}{2}$ and $-\frac{1}{2}$. Its period is still 360°

2. (a) This time, you are going to **sketch** the graph of **$y = 5\sin x°$**.

step 1
draw a set of x and y-axes.

step 2
draw the "sine-shaped" curve.

step 3
now fill in the 0 to 360 and +5/–5.

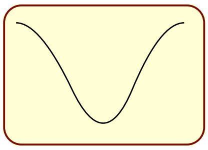


(b) What is the **maximum** value and what is the **minimum** value the graph attains ?

(c) What is the **amplitude** and **period** of **$y = 5\sin x°$** ?

3. (a) Sketch the graph of **$y = 10\cos x°$**, $\{0 \le x \le 360\}$.

- remember - axes first, then cosine-shaped graph and **lastly** the scales.



(b) What are the **maximum-minimum** values ?

(c) What is the amplitude and the period of **$y = 10\cos x°$** ?

4. Make neat sketches of the following trig. graphs, using the x–values - $\{0 \le x \le 360\}$.

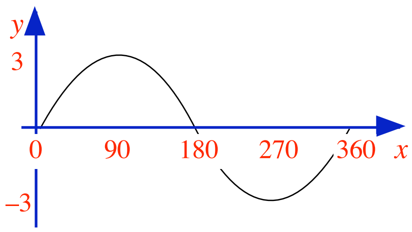
(a) $y = 8\sin x°$. (b) $y = 60\sin x°$.

(c) $y = 4\cos x°$. (d) $y = 0{\cdot}65\cos x°$.

(e) $y = \frac{1}{2}\sin x°$. (f) $y = 5\tan x°$.

5. This time you are going to draw the graph of the function **$y = –3\sin x°$**.

Remember that this is a sketch of $y = 3\sin x°$.



(a) How do you think $y = –3\sin x°$ will differ ?

(b) What are the **maximum-minimum** values ?

(c) What is the amplitude and the period of **$y = –3\sin x°$** ?

6. Make a neat labelled sketch of :-

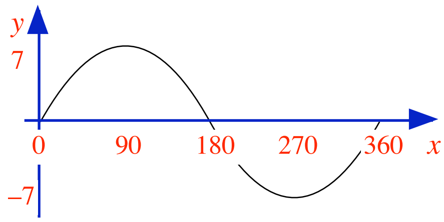
(a) $y = –2\cos x°$

(b) $y = –0{\cdot}5\sin x°$

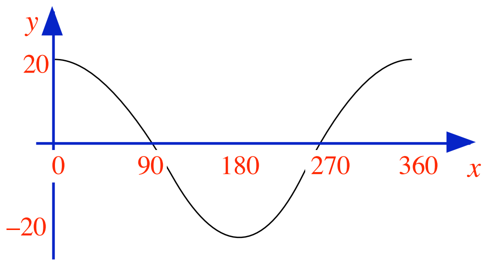
7. Each of the following trig graphs represents a function of the form **$y = a\sin x°$** or **$y = a\cos x°$**.

Write down the equation of each function.

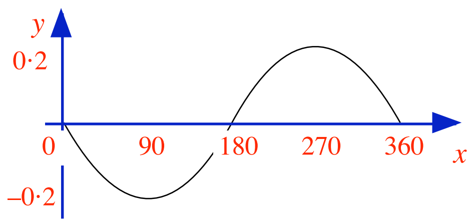
(a)



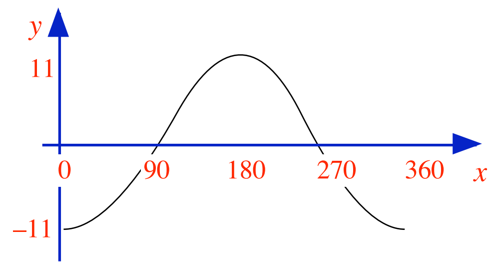
(b)



(c)



(d)



More Sine and Cosine Functions ($y = \sin ax°$ and $y = \cos ax°$)

$y = \sin 2x°$

We are going to study the $y = \sin 2x°$ for various values of x.

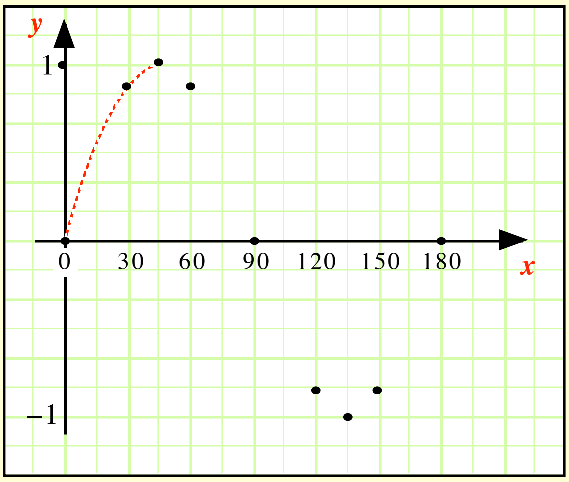
for example, when $x = 30$ => $2x = 60$ => $\sin 2x° = \sin 60° = 0·87$

(a) Copy this table and use your calculator to complete it (to 2 decimal places each time).

x	0	30	45	60	90	120	135	150	180
$\sin 2x°$	0	0·87	1·00	0·87	0	–0·87	...	...	...

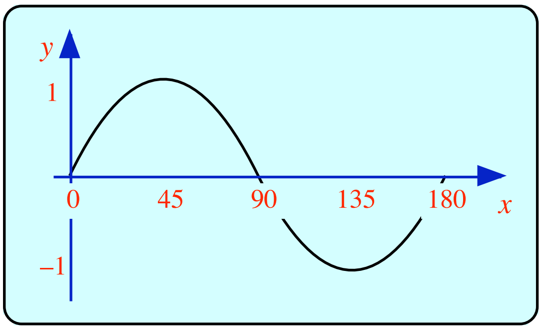
(b) Draw a set of axes on squared paper, plot the above nine points and join them up with a smooth curve.

Show your graph of $y = \sin 2x°$ to your teacher.



Exercise 7·5

1. (a) This time, just sketch the graph $y = \sin 2x°$ on a smaller scale like this :-



(b) What is the **highest** value the graph attains ?

(c) What is its **lowest** value ?

(d) What is the **amplitude** and the **period** of $y = \sin 2x°$?

(e) For what values of "x" does the curve cut the x-axis ?

You should have noticed the following :-

- the graph is **identical** in shape to that of $y = \sin x°$.
- its **maximum** and **minimum** values are still +1 and –1, and its **amplitude** is still 2.
- its period is no longer 360° – its period is now 360° ÷ **2** = **180°**.

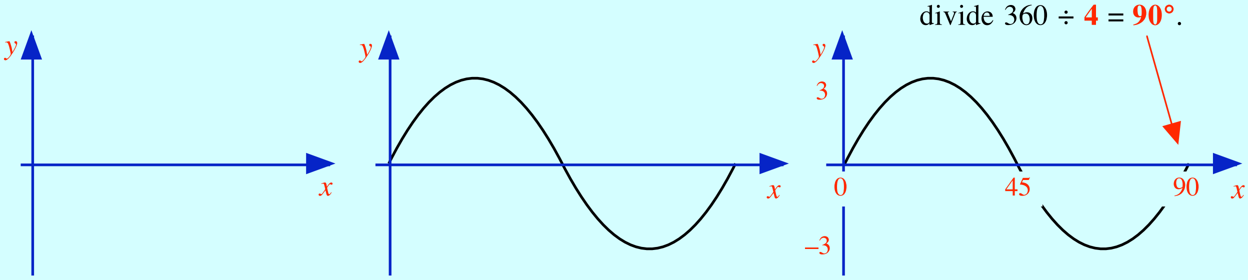
You will find that :-
the period of $y = \sin 3x°$ has a period of 360 ÷ **3** = **120°**
the period of $y = \sin 10x°$ has a period of 360 ÷ **10** = **36°**
the period of $y = \cos 4x°$ has a period of 360 ÷ **4** = **90°**
the period of $y = \tan 2x°$ has a period of 180 ÷ **2** = **90°**

2. (a) This time, you are going to **sketch** the graph of **$y = 3\sin 4x°$**.

step 1
draw a set of x and y-axes.

step 2
draw the "sine-shaped" curve.

step 3
to find the **period** :-
divide 360 ÷ **4** = **90°**.

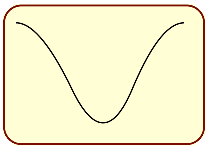


(b) What is the **maximum** value and what is the **minimum** value of the function ?

(c) What is the **period** of **$y = 3\sin 4x°$** ?

3. (a) Sketch the graph of **$y = \cos 3x°$**.

- remember - axes first, then cosine-shaped graph and **lastly** the scales.



(b) What are the **maximum-minimum** values ?

(c) What is the period of **$y = \cos 3x°$** ?

4. Make neat sketches of the following trig. graphs.

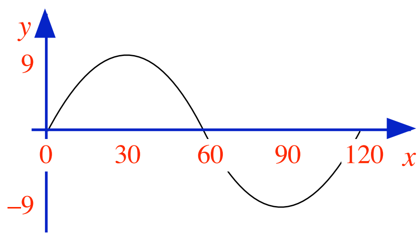
(a) $y = 6\sin 3x°$. (b) $y = 50\sin 6x°$.

(c) $y = 5\cos 2x°$. (d) $y = 0{\cdot}7\cos 4x°$.

(e) $y = 12\sin\frac{1}{2}x°$. (f) $y = 5\tan 2x°$. ***(careful)***

5. This time you are going to draw the graph of the function **$y = –9\sin 3x°$**.

Remember that this is a sketch of $y = 9\sin 3x°$.



(a) How do you think $y = –9\sin 3x°$ will differ ?

(b) What are the **maximum-minimum** values ?

(c) What is the period and amplitude of **$y = –9\sin 3x°$** ?

6. Make a neat labelled sketch of **1 cycle** of :-

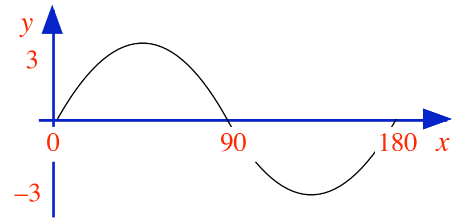
(a) $y = –12\cos 5x°$ (b) $y = –0{\cdot}2\sin 6x°$

(c) $y = –0{\cdot}1\sin\frac{1}{2}x°$ (d) $y = –\frac{1}{8}\cos 30x°$

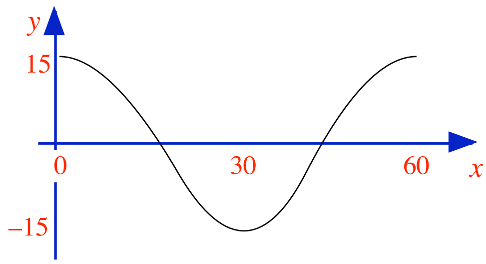
7. Each of the following trig graphs represents a function of the form **$y = a\sin bx°$** or **$y = a\cos bx°$**.

Write down the equation of each function.

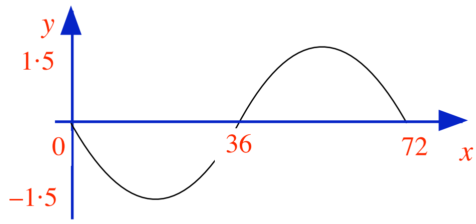
(a)



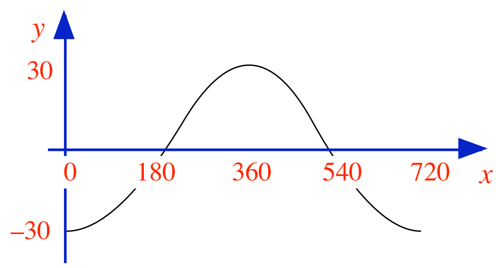
(b)



(c)



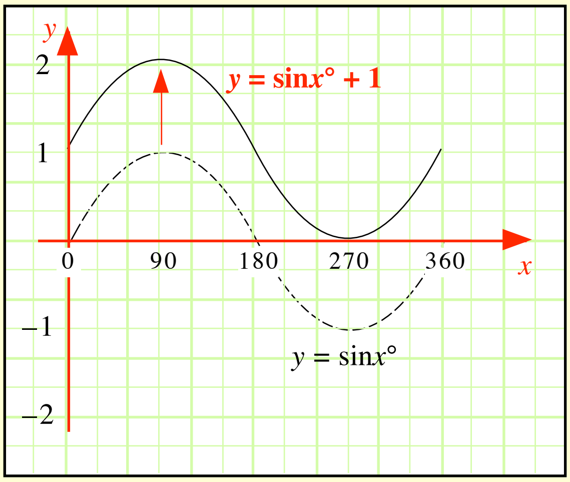
(d)



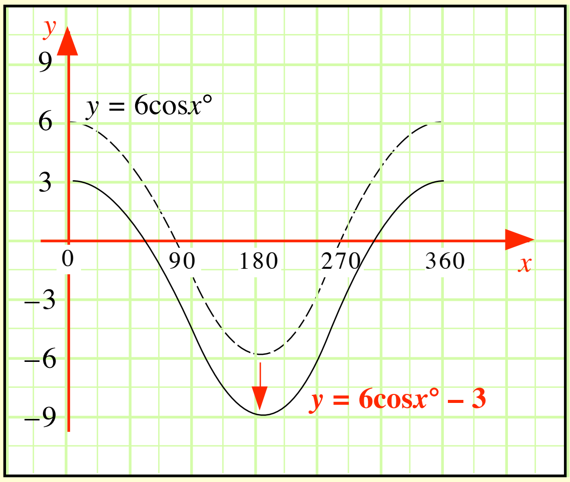
Yet more Sine and Cosine Functions ($y = a\sin x° + b$ and $y = a\cos x° + b$)

When the sine or cosine function has a number added on (or subtracted), the simple effect is to "slide" the basic sine or cosine function upwards (or downwards) by that amount.

Example 1 :- $y = \sin x° + 1$



Example 2 :- $y = 6\cos x° – 3$



Note :-
- The **period** of the new function with the added (or subtracted) term remains **the same**.
- The **amplitude** stays the same (*the difference between the highest and lowest points*).
- But the **maximum** and the **minimum** values change.

  In **example 1**, the maximum and minimum changes from 1 and –1 —> to 2 and 0.

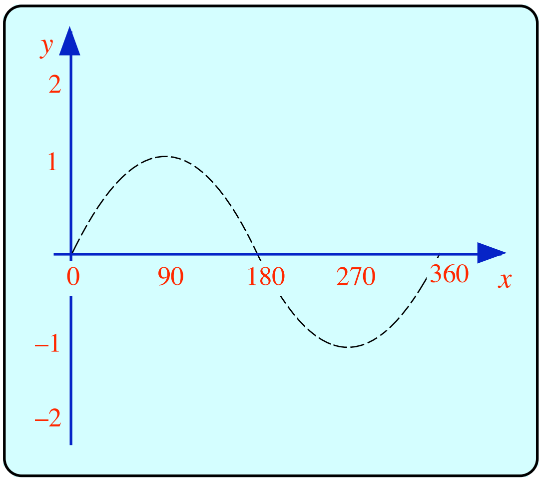
  In **example 2**, the maximum and minimum changes from 6 and –6 —> to 3 and –9.
- To draw $y = \sin x + 2$, simply sketch the graph of $y = \sin x$ and **move each point up by 2**.

Exercise 7·6

1. Shown below is a sketch of the function

   $y = \sin x°$.

   (a) Make a neat copy of the graph, showing the graph dotted as in the sketch.



   (b) On your drawing, show also the graph of the function :- $y = \sin x° – 1$.

2. (a) Make a neat (dotted) sketch of the function

   $y = \cos x°$,

   showing all the main features and values.

   (b) On the same graph, show the function :-

   $y = \cos x + 2$.

3. (a) This time, make a sketch showing $y = 4\sin x°$.

   (b) On the same graph, show the function :-

   $y = 4\sin x – 2$,

   showing all its main features.

4. (a) This time, sketch the graph of the function :-

   $y = 6\cos x°$,

   (b) On the same graph, show $y = 6\cos x + 3$,

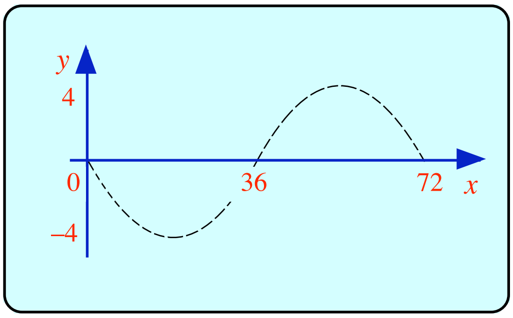
   indicating all the main features and values.

5. Make neat sketches of each of the following, showing all the main features and values. (*hint :- sketch the "basic" trig function first*).

(a) $y = 2\sin x° + 2$ (b) $y = \cos x° - 3$

(c) $y = 40\sin x° - 40$ (d) $y = 12\cos x° - 6$

6. Shown below is the graph of $\mathbf{y = -4\sin 5x°}$.



(*remember how it is "**upside-down**"*).

Make a neat sketch of this (dotted) curve, and show on it the graph of $\mathbf{y = -4\sin 5x° + 4}$.

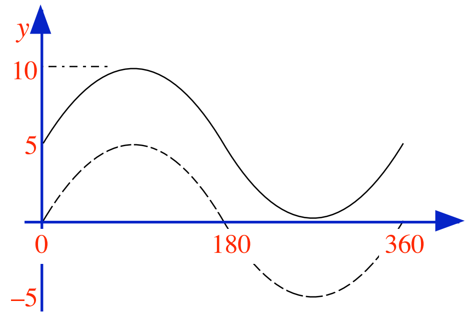
7. Sketch the graph of $\mathbf{y = -6\cos x°}$, (*dotted*), and show also the graph of $\mathbf{y = -6\cos x° - 3}$, indicating all of its main features and values.

8. Make neat sketches of each of the following, showing all the main features and values.

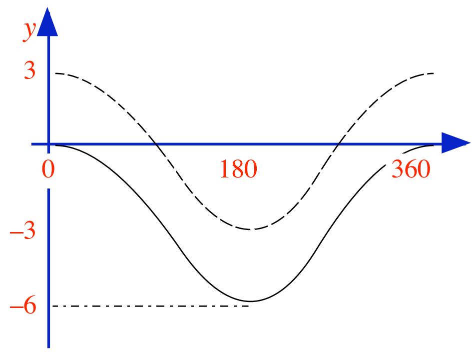
(a) $y = -3\sin x° + 3$ (b) $y = -\cos x° - 2$

(c) $y = 10 - 10\sin x°$ (d) $y = -1 - 2\cos x°$

9. Work out the equation of the following trig function from its graph :-

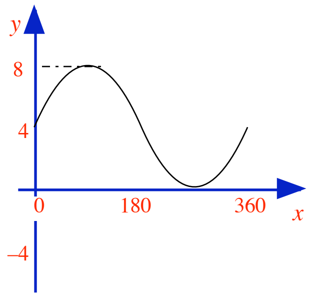


10. Determine the equation of this trig graph :-

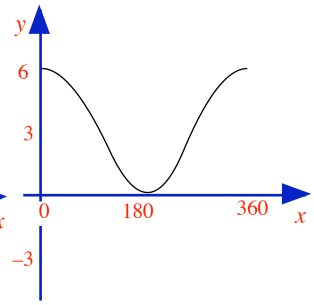


11. Determine the equation of each of the following trig functions from their graphs :-

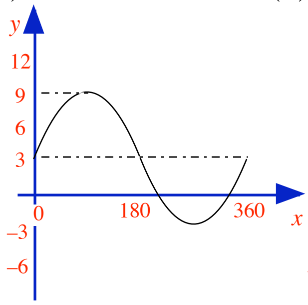
(a)



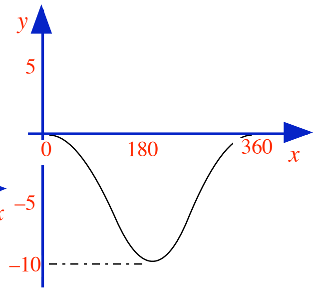
(b)



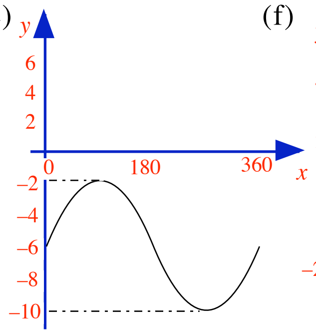
(c)



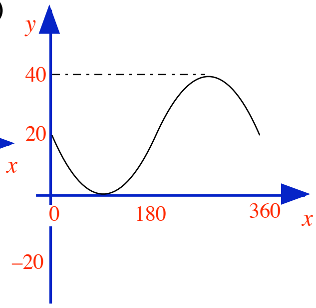
(d)



(e)



(f)

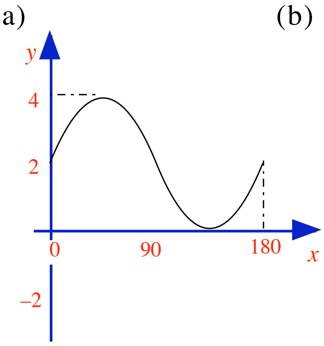


12. **Harder** - Each of these functions is of the form :-

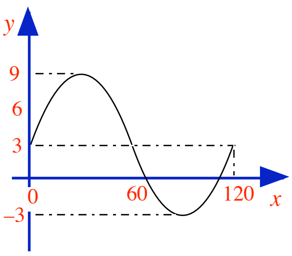
$$y = a\sin \underline{b}x + c.$$

Determine the equation of each.

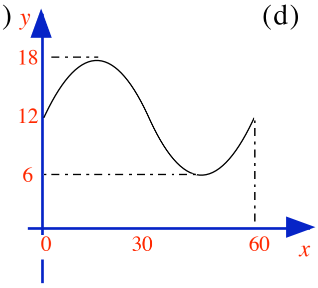
(a)



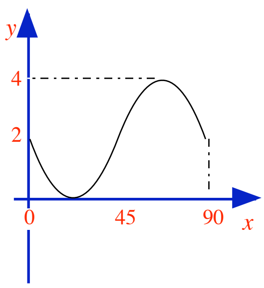
(b)



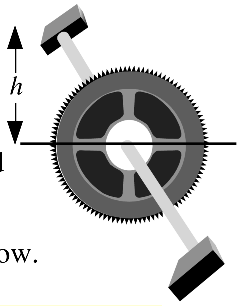
(c)



(d)

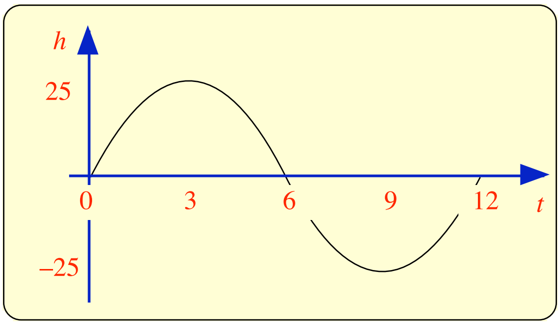


13. The pedal on this bicycle crank is rotated.



Its height above (and below) the centre of the shank is noted as the pedal rotates.

This is shown on the graph below.



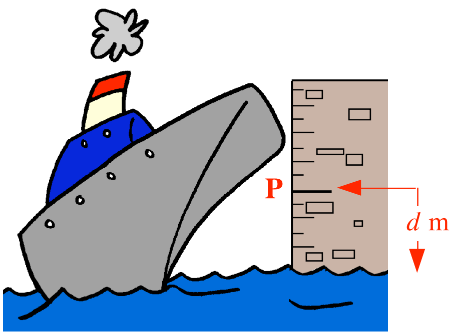
- t represents the time (in seconds)
- h represents the height (in centimetres).

(a) What is the **period** of the graph ?
(*this is the time taken for 1 rotation*).

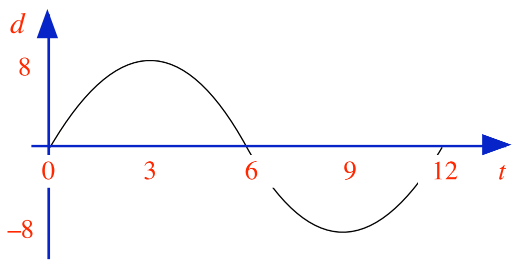
(b) Write down the equation of the graph :-

$$h = \ldots.\sin \ldots.t^\circ.$$

14. The water level rises and falls every 12 hours in a harbour as the tide comes in and out.



A graph, showing the depth (d m) of water measured from point **P**, is shown below.
The time (t) is measured in hours.

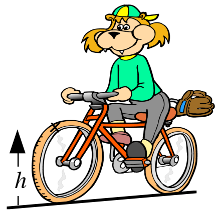


(a) How high above P is the water at high tide ?

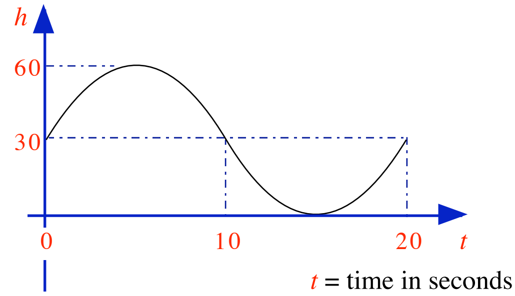
(b) Write down the equation of the graph :-

$$d = \ldots.\sin \ldots.t^\circ.$$

15. A chalk-mark is made on the tyre of a bicycle wheel.



As the wheel rotates along the ground, its height in centimetres is recorded and shown on the graph below.



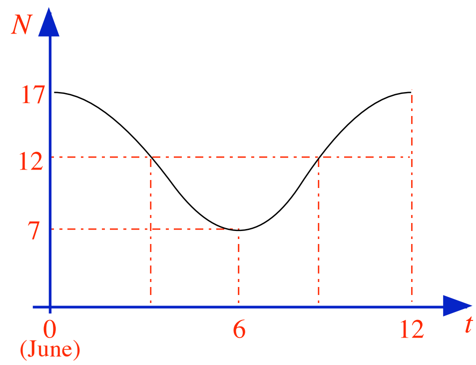
(a) From the graph, say what the diameter of the wheel must be.

(b) What is the period of the graph ?

(c) Write down the equation of the graph :-

$$h = \ldots.\sin \ldots.t^\circ + \ldots. \,.$$

16. The graph below shows the average number of hours of daylight, (*each day*), there is throughout the year starting from the month of June.



- N is the number of hours sunshine daily
- t is the number of months after June.

(a) What is the maximum number of hours of sunshine each day, and in which month ?

(b) What is the minimum number, and when ?

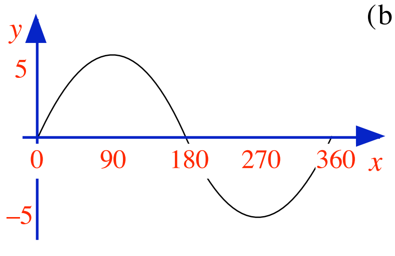
(c) The **equinoxes** (*Spring and Autumn*) are when there is an equal number of hours of light and dark. Which months ?

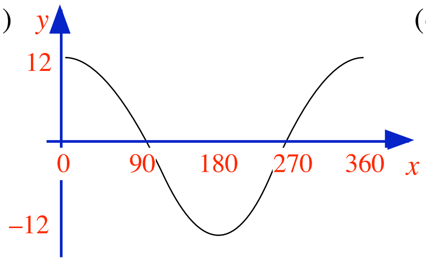
(d) Find the equation of the graph in the form :-

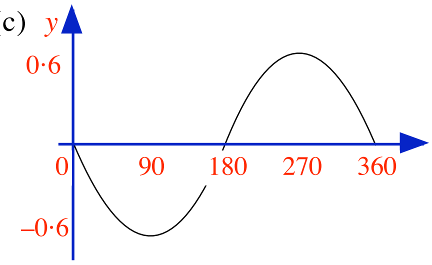
$$N = \ldots.\cos \ldots.t^\circ + \ldots. \,.$$

1. Sketch the sine graph, the cosine graph and the tangent graph, $\{0 \leq x \leq 360\}$, on different diagrams, indicating the shape of each and all the important points through which they pass.

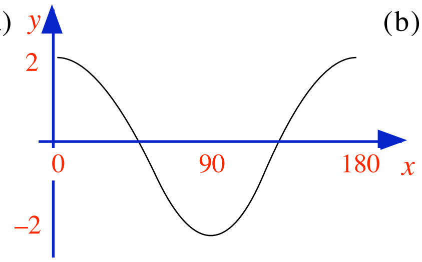
2. Write down the equations of the following graphs :-

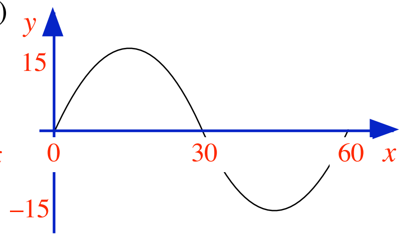
(a)


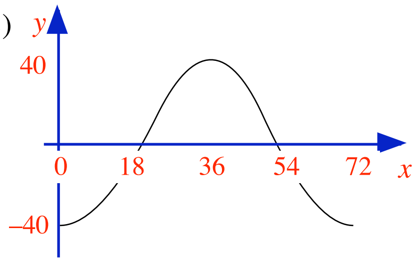
(b)


(c)


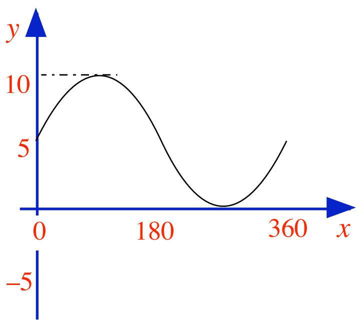
3. Write down the equations of the following graphs :-

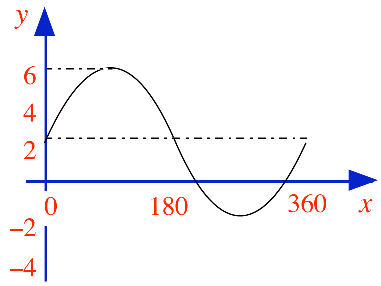
(a)


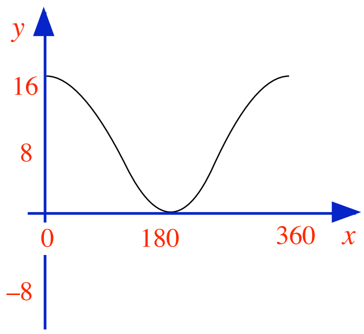
(b)


(c)


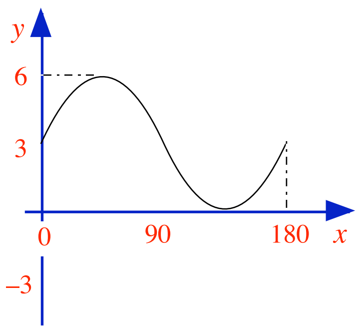
4. Write down the equations of the following graphs :-

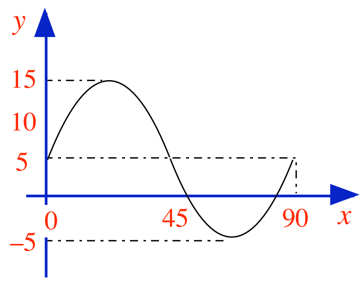
(a)


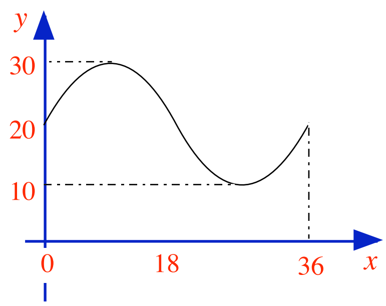
(b)


(c)


5. Write down the equations of the following graphs :-

(a)


(b)


(c)


6. Make a neat sketch of each function, showing the shape, scale and important points on your graphs.

(a) $y = 25\sin x°$ (b) $y = 10\cos x°$ (c) $y = \sin 3x°$

(d) $y = 8\cos 4x°$ (e) $y = \sin x° + 1$ (f) $y = 2\sin x° - 2$

(g) $y = 20\cos x° + 10$ (h) $y = -2\sin 6x°$ (i) $y = 4\cos 2x° - 4$