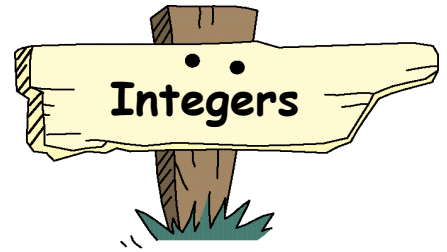


# Chapter 1



## Definition

An **INTEGER** is simply a **POSITIVE** or a **NEGATIVE whole number**.  
(0 is also included in the set of integers)

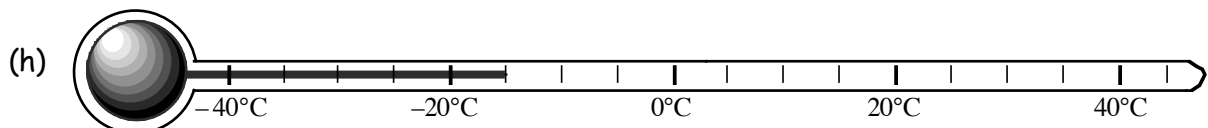
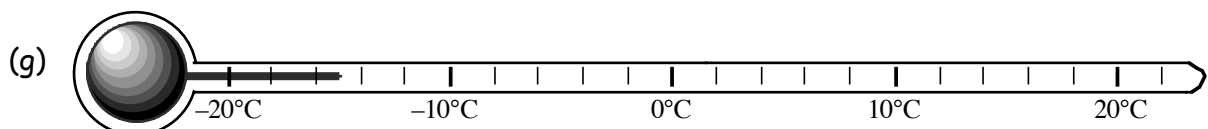
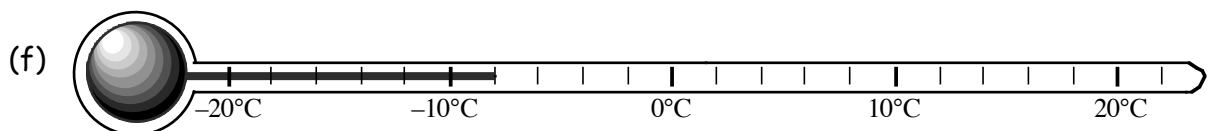
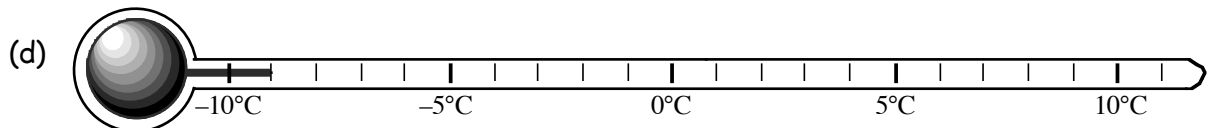
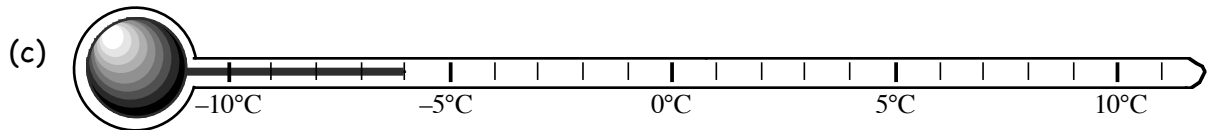
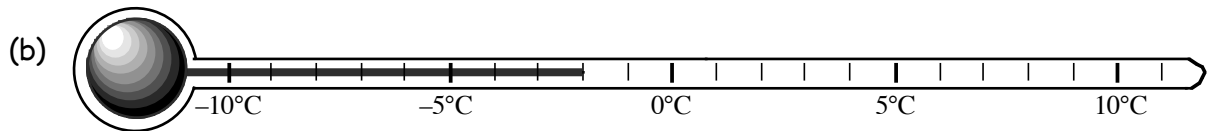
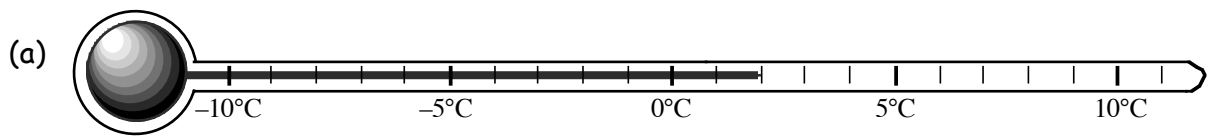
## Examples :-

-7, -19, 8, 23, 0, -5, 1000, -2003, etc. are all **INTEGERS**.

$2.5$ ,  $\frac{1}{2}$ ,  $-2.1$ ,  $1\frac{3}{4}$ ,  $-13.6$ , etc are **NOT** integers

## Exercise 1

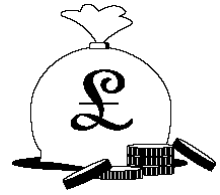
1. A thermometer is the most obvious place to see positive and negative numbers (integers). What temperatures are shown here :-



2. Negative numbers also occur when considering how much money you have (or don't have !) in a bank.

If you have £20 in your bank account, the computer notes this as

+£20.00



- (a) If you are "overdrawn" by £20, what do you think the computer stores this as ?  
 (b) State what each of the following "bank balances" mean, in real terms :-

(i) 

22/10/02
balance + £63.50

(ii) 

15/11/01
balance - £18.00

(iii) 

16/04/02
balance - £123.50

(iv) 

20/12/99
balance £0.00

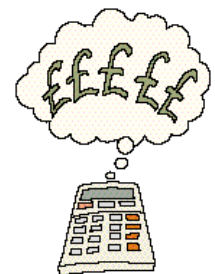
- (c) I had £15 in my bank account and withdrew £20.  
 What will the computer show my balance to be now ?



- (d) My bank balance is shown opposite.  
 I pay £10 into my account.  
 What will my new balance be ?

balance - £35.00

- (e) My bank balance was exactly £0.00.  
 I withdrew £80.  
 What will my new balance be ?



- (f) Last week my bank balance stood at - £40.00.  
 I withdrew a further £20.  
 What will my balance be now ?

- (g) If my bank balance stood at - £85, how much must I deposit to "clear my overdraft" ?  
 (h) My balance showed +£15.50.  
 I signed two cheques, one for £18.20 and another for £7.90.  
 What will my new balance now show ?

3. When heights are measured as being above or below sea level, we can use negative numbers to describe them.

Heights **ABOVE** sea level are **positive** (+)

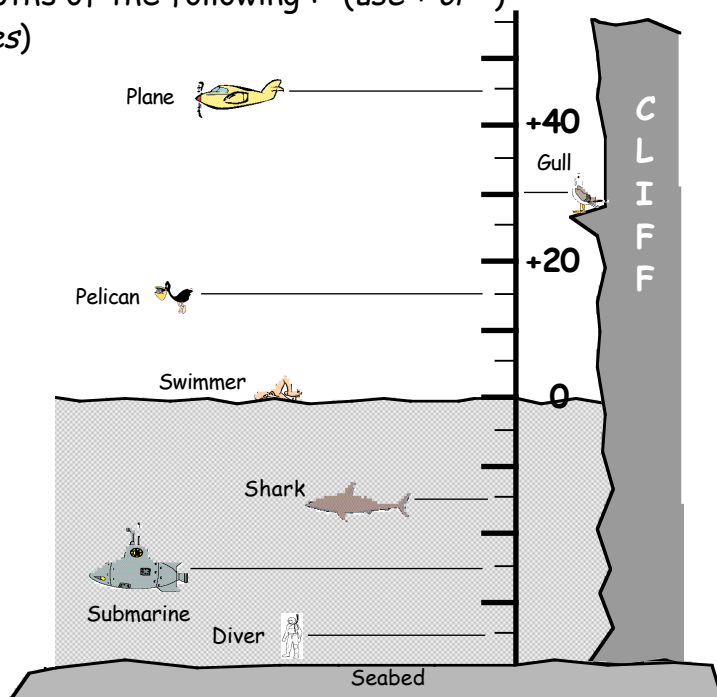
Heights **BELOW** sea level are **negative** (-)

cont'd .....

(a) Write down the heights or depths of the following :- (use + or -)  
(all measurements are in metres)

- (i) the gull
- (ii) the pelican
- (iii) the shark
- (iv) the cliff top
- (v) the diver
- (vi) the sea bed
- (vii) the plane
- (viii) the submarine

(b) How high is the pelican above the shark ?



4. A fourth use of negative numbers is in the context of **TIME**.

We say we live in the year 2003 A.D. (anno domini)

This means 2003 years since the birth of Christ. (or +2003)



If someone lived in the year 50 B.C. (before Christ), we say they lived in the year (-50).

(a) Write the following dates using the "+" or "-" signs :-

- (i) 1066 A.D.
- (ii) 1812 A.D.
- (iii) 25 B.C.
- (iv) 1200 B.C.

(b) Artimus was born in 106 A.D. and died in 148 A.D.

How old was he when he died ?



(c) Maximus Plonktus was born in 82 B.C. and died in 22 B.C.

What was his age when he died ?



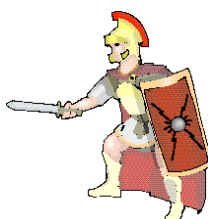
(d) Divinius Minumus was born in 15 B.C. and died in 35 A.D.

How old was he on his death ?



(e) Sanctius Catcus died in 24 A.D. at the age of 49.

In which year was he born ?



## Studying Integers

The easiest way to handle integers is to draw or imagine them as temperatures on a thermometer.

### Exercise 2 (No calculator)

- Use a ruler to copy this thermometer neatly into your jotter. (it does not have to go all the way from  $-24$  to  $+24$ ).
- Look at your thermometer.

What is the temperature that is :-

- |  |   |
|--|---|
| (a) $5^{\circ}\text{C}$ up from $8^{\circ}\text{C}$ ?    | (b) $9^{\circ}\text{C}$ up from $0^{\circ}\text{C}$ ?     |
| (c) $12^{\circ}\text{C}$ up from $6^{\circ}\text{C}$ ?   | (d) $6^{\circ}\text{C}$ down from $11^{\circ}\text{C}$ ?  |
| (e) $9^{\circ}\text{C}$ down from $20^{\circ}\text{C}$ ? | (f) $4^{\circ}\text{C}$ up from $-2^{\circ}\text{C}$ ?    |
| (g) $7^{\circ}\text{C}$ down from $-3^{\circ}\text{C}$ ? | (h) $20^{\circ}\text{C}$ up from $-6^{\circ}\text{C}$ ?   |
| (i) $6^{\circ}\text{C}$ down from $5^{\circ}\text{C}$ ?  | (j) $10^{\circ}\text{C}$ down from $0^{\circ}\text{C}$ ?  |
| (k) $4^{\circ}\text{C}$ down from $-5^{\circ}\text{C}$ ? | (l) $12^{\circ}\text{C}$ down from $-2^{\circ}\text{C}$ ? |
| (m) $2^{\circ}\text{C}$ up from $-10^{\circ}\text{C}$ ?  | (n) $15^{\circ}\text{C}$ up from $-20^{\circ}\text{C}$ ?  |

- Can you see that  $4^{\circ}\text{C}$  is " **$5^{\circ}\text{C}$  up from**"  $-1^{\circ}\text{C}$  ?

Copy and complete these in the same way :-

(say whether it's " .. up from" or " .. down from" each time)

- |   |   |
|---|---|
| (a) $12^{\circ}\text{C}$ is ..... from $4^{\circ}\text{C}$    | (b) $6^{\circ}\text{C}$ is ..... from $11^{\circ}\text{C}$    |
| (c) $0^{\circ}\text{C}$ is ..... from $15^{\circ}\text{C}$    | (d) $4^{\circ}\text{C}$ is ..... from $-3^{\circ}\text{C}$    |
| (e) $-7^{\circ}\text{C}$ is ..... from $0^{\circ}\text{C}$    | (f) $2^{\circ}\text{C}$ is ..... from $-10^{\circ}\text{C}$   |
| (g) $-15^{\circ}\text{C}$ is ..... from $-10^{\circ}\text{C}$ | (h) $-4^{\circ}\text{C}$ is ..... from $4^{\circ}\text{C}$    |
| (i) $20^{\circ}\text{C}$ is ..... from $-20^{\circ}\text{C}$  | (j) $-50^{\circ}\text{C}$ is ..... from $-40^{\circ}\text{C}$ |

- In Moscow, the temperature was  $-25^{\circ}\text{C}$ .  
In Stalingrad it was  $15^{\circ}$  colder.

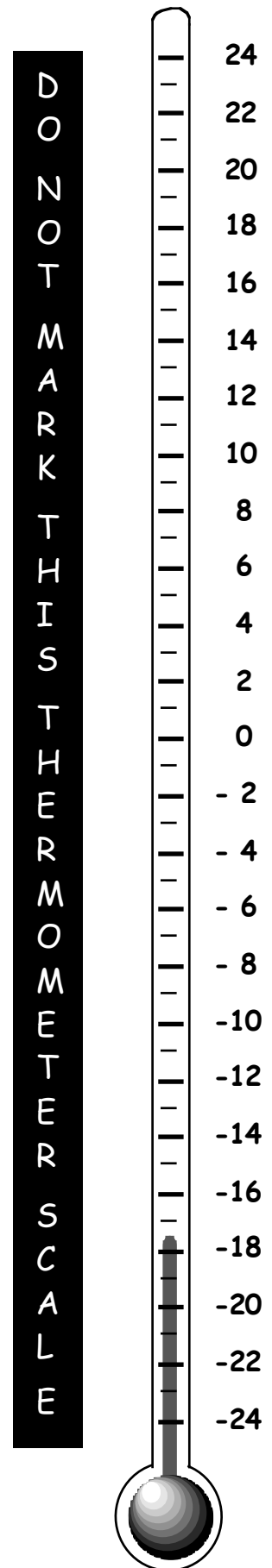
What was the temperature in Stalingrad ?



- 



The temperature in the Sahara desert dropped from  $+35^{\circ}\text{C}$  at noon to  $-15^{\circ}\text{C}$  at midnight.  
By how much had the temperature dropped ?

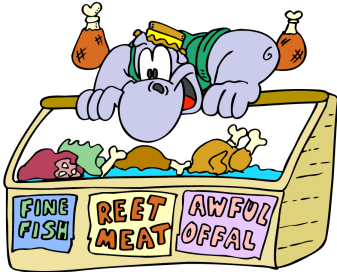


6. When I left my freezer door open for an hour, the temperature rose from  $-22^{\circ}\text{C}$  to  $-13^{\circ}\text{C}$ .

By how much had the temperature risen ?



7.



When a butcher put a side of beef in his freezer, its temperature fell by a **steady amount** each hour.

It started at  $12^{\circ}\text{C}$  and fell to  $7^{\circ}\text{C}$  in one hour.

What would the temperature be after :-

- (a) 2 hours ?                      (b) 3 hours ?  
 (c) 4 hours ?                      (d) 10 hours ?

### Adding and Subtracting Integers

When adding and subtracting integers, the best way is to draw or imagine them as temperatures on a thermometer.

**Example 1** :- To find  $(3) + 7$ ,  
 imagine the 3 on a thermometer.

To do the "+7" bit you go UP by 7

$$\rightarrow 3 + 7 = 10$$

(+ 7)

(3)

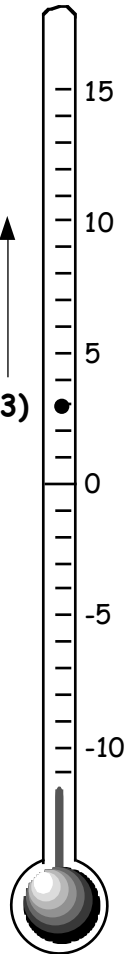
**Example 2** :- To find  $4 + (-6)$ ,  
 imagine the 4 on a thermometer.

To do the "+(-6)" bit you go DOWN by 6  $\rightarrow 4 + (-6) = -2$

**Example 3** :- To find  $(4) - 7$ ,  
 imagine the 4 on a thermometer.

To do the "-7" bit you go DOWN by 7

$$\rightarrow 4 - 7 = -3$$



### **Exercise 3** (No calculator)

1. Use the thermometer which you drew from the last exercise, (or draw a new one), to help you here.

Write down each question first, then the answer :-

- |                   |                    |                 |                    |
|-------------------|--------------------|-----------------|--------------------|
| (a) $5 + 7$       | (b) $3 + 8$        | (c) $0 + 9$     | (d) $8 + (-2)$     |
| (e) $6 + (-1)$    | (f) $12 + (-12)$   | (g) $4 + (-5)$  | (h) $2 + (-10)$    |
| (i) $0 + (-9)$    | (j) $(-3) + 6$     | (k) $(-5) + 5$  | (l) $(-1) + 17$    |
| (m) $(-10) + 6$   | (n) $(-15) + 11$   | (o) $2 + (-7)$  | (p) $(-2) + (-7)$  |
| (q) $(-4) + (-4)$ | (r) $(-5) + (-10)$ | (s) $(-12) + 2$ | (t) $(-12) + (-2)$ |

2. Again use your thermometer to help here :-

(remember :-  $8 - 9$  means "go to 8, then move down by 9").

- |                |                  |                 |                  |
|----------------|------------------|-----------------|------------------|
| (a) $9 - 3$    | (b) $10 - 10$    | (c) $4 - 1$     | (d) $3 - 5$      |
| (e) $5 - 10$   | (f) $2 - 12$     | (g) $0 - 15$    | (h) $(-1) - 4$   |
| (i) $(-7) - 3$ | (j) $(-11) - 5$  | (k) $(-1) - 21$ | (l) $0 - 35$     |
| (m) $19 - 39$  | (n) $(-15) - 25$ | (o) $100 - 300$ | (p) $(-71) - 29$ |

3. A **Mixture** !! The rule is simple.

Picture the first number on your thermometer.

If you add a positive number move **UP**.

If you add a negative number or take away a number move **DOWN**.

- |                 |                 |                  |                     |
|-----------------|-----------------|------------------|---------------------|
| (a) $3 + 6$     | (b) $2 + (-5)$  | (c) $4 - 7$      | (d) $(-3) + 10$     |
| (e) $-2 + (-3)$ | (f) $8 - 12$    | (g) $(-2) - 5$   | (h) $(-20) + (-15)$ |
| (i) $-15 + 20$  | (j) $0 - 13$    | (k) $0 + (-13)$  | (l) $(-15) + (-7)$  |
| (m) $15 + (-7)$ | (n) $(-15) + 7$ | (o) $(-11) + 11$ | (p) $63 - 97$       |

### Simple Multiplication and Division of Integers

Since  $3 \times 5 = 15$ , then obviously  $3 \times (-5)$  cannot also be 15.

$3 \times (-5)$  means "3 lots of -5" = -15.

**Some Examples** :-  $2 \times (-7) = -14$        $5 \times (-8) = -40$ .  
 $(-3) \times 6 = -18$        $(-10) \times 10 = -100$ .

Similarly :- since  $10 \div 2 = 5$  then obviously  $(-10) \div 2$  cannot also be 5.  
 $(-10) \div 2 =$  "-10 shared by 2" = -5.

**Some Examples** :-  $(-12) \div 4 = -3$        $(-20) \div 10 = -2$ .  
 $(-36) \div 6 = -6$        $(-100) \div 20 = -5$ .

### **Exercise 4** (No calculator)

1. Write down each of the following and find the answers :-

- |                     |                     |                      |                      |
|---------------------|---------------------|----------------------|----------------------|
| (a) $4 \times (-5)$ | (b) $6 \times (-7)$ | (c) $2 \times (-9)$  | (d) $5 \times (-5)$  |
| (e) $(-8) \times 3$ | (f) $(-9) \times 4$ | (g) $(-11) \times 2$ | (h) $(-10) \times 7$ |
| (i) $6 \times (-8)$ | (j) $8 \times (-3)$ | (k) $4 \times (-12)$ | (l) $7 \times (-7)$  |
| (m) $9 \times (-1)$ | (n) $(-9) \times 3$ | (o) $(-2) \times 10$ | (p) $(-9) \times 5$  |

2. Write down each of the following and find the answers :-

- |                    |                     |                    |                     |
|--------------------|---------------------|--------------------|---------------------|
| (a) $(-30) \div 6$ | (b) $(-20) \div 5$  | (c) $(-56) \div 7$ | (d) $(-63) \div 9$  |
| (e) $(-40) \div 2$ | (f) $(-90) \div 10$ | (g) $(-33) \div 3$ | (h) $(-32) \div 4$  |
| (i) $(-8) \div 8$  | (j) $(-5) \div 1$   | (k) $(-54) \div 6$ | (l) $(-100) \div 5$ |

3. Find the answers to the following :-

- (a)  $(4 \times 9) \div 6$       (b)  $(2 \times (-10)) \div 5$       (c)  $3 \times (-2) \times 4$       (d)  $5 \times (-1) \times 6$   
 (e)  $3 \times (-8) \div 6$       (f)  $(-6) \times 6 \div 4$       (g)  $6 \times (-4) \div 2$       (h)  $10 \times (-10) \div 5$

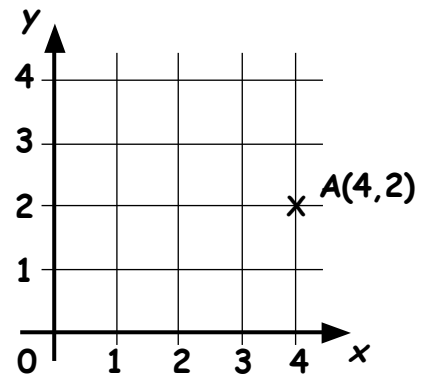
4. Find the following :- (hint : find the bit in brackets first)

- (a)  $(8 + (-5)) \times 7$       (b)  $6 \times (4 - 7)$       (c)  $((-10) + 2) \times 2$   
 (d)  $((-4) - 8) \div 2$       (e)  $10 \times (12 - 14)$       (f)  $(8 - 3) \times (-5)$   
 (g)  $((-3) - 4) \times 5$       (h)  $(6 + (-12)) \div 3$       (i)  $((-9) - 11) \div 5$   
 (j)  $(-4) \times ((-2) + 7)$       (k)  $(8 + (-8)) \times 5$       (l)  $(-60) + (-30) \div 10$

## Coordinates

**Revision :-** You should know what a Cartesian diagram (or a co-ordinate diagram) looks like.

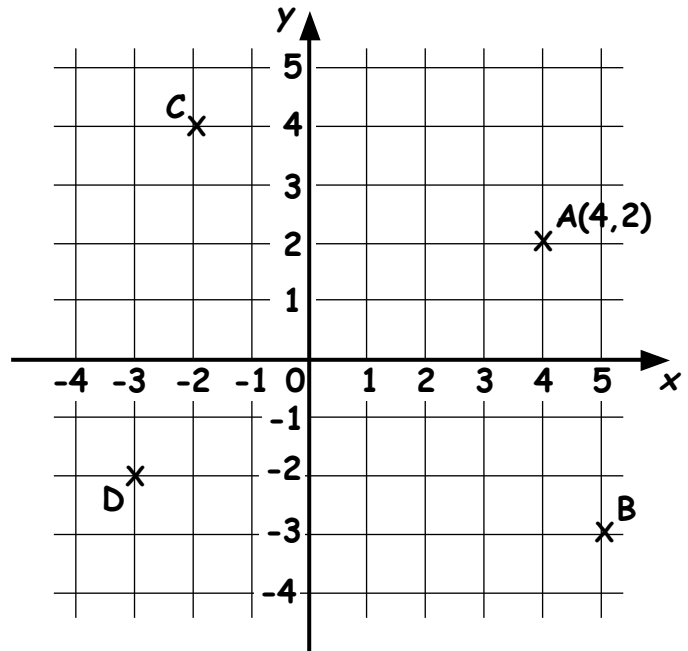
**Remember :-** x-axis (or horizontal axis).  
 y-axis (or vertical axis).  
 the origin (O).  
 A is 4 (right) and 2(up) from the origin.  
 => A(4,2) has x-coordinate 4 and y-coordinate 2.



We now extend the set of x and y axes backwards and downwards.

Look at the numbers on the x- and y- axes.

They now include **NEGATIVE** values.



Can you see the following :-

the point B is 5 (to the right) and 3 (down) from the origin → B(5, -3)

the point C is 2 (to the left) and 4 (up) from the origin → C(-2, 4)

the point D is 3 (to the left) and 2 (down) from the origin → D(-3, -2) ?

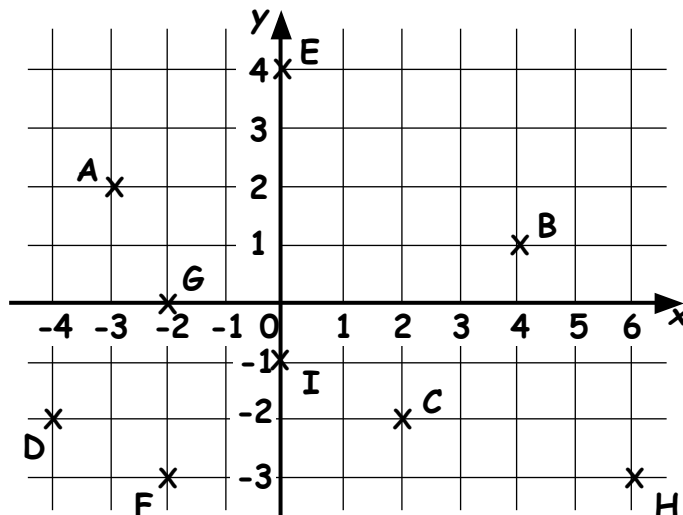
## Exercise 5

1. Look at this coordinate diagram.

The coordinates of A are

$$A(-3, 2)$$

Write down the coordinates of the other 8 points.

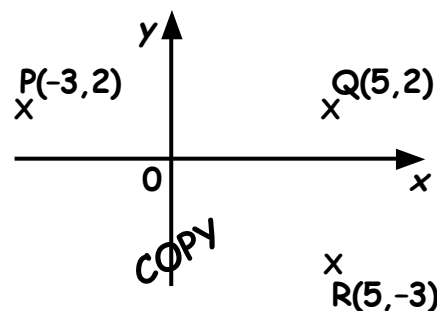


2. Draw a large set of axes (-10 to 10 on both scales).

Plot each set of points, join them up and state what shape each is :-

- (a)  $A(3,3)$   $B(5,4)$   $C(7,3)$   $D(5,-2)$ .      (b)  $E(-7,5)$   $F(-5,8)$   $G(2,8)$   $H(0,5)$ .  
 (c)  $I(-10,3)$   $J(-8,3)$   $K(-9,-3)$ .      (d)  $L(1,-5)$   $M(-4,-4)$   $N(-5,1)$   $O(0,0)$ .  
 (e)  $P(4,-5)$   $Q(6,-7)$   $R(5,-9)$   $S(3,-9)$   $T(2,-7)$   $P(4,-5)$ .  
 (f)  $U(-8,-3)$   $V(-6,-3)$   $W(-5,-5)$   $X(-6,-7)$   $Y(-8,-7)$   $Z(-9,-5)$   $U(-8,-3)$ .

3. (a) Copy this diagram and plot the three points  $P(-3,2)$ ,  $Q(5,2)$  and  $R(5,-3)$ .



- (b) Try to find a 4th point, (call it S) such that PQRS is a rectangle. Show S on your diagram, and write down its coordinates.

4. (a) Draw a set of axes, (-6 to 6 on both scales) and plot the four points  $A(2,1)$ ,  $B(3,5)$ ,  $C(5,5)$ ,  $D(6,1)$ .

- (b) Join the four points and state which type of shape is formed.

- (c) "Flip" each of the four points over the x-axis to form a new four-sided shape. (This is called "**REFLECTING**" the shape).

- (d) Write down the coordinates of the four corners of this new reflected shape.

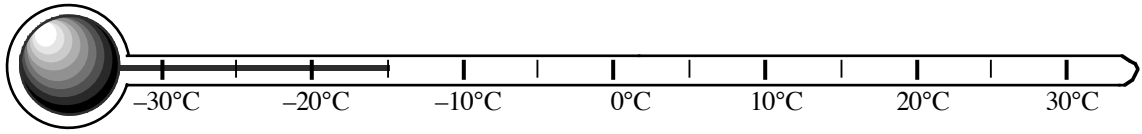
5. Draw a set of axes, (-5 to 5 on both scales). Join each of these sets of points with lines :-

- |                          |                          |                          |                          |
|--------------------------|--------------------------|--------------------------|--------------------------|
| $(2, 3)$ to $(4, 3)$     | $(1, 0)$ to $(1, -2)$    | $(-2, 0)$ to $(-4, 0)$   | $(-1, -3)$ to $(-1, -5)$ |
| $(-1, 2)$ to $(1, 2)$    | $(-3, 1)$ to $(-3, 3)$   | $(-1, -5)$ to $(-3, -5)$ | $(2, 1)$ to $(2, 3)$     |
| $(2, 0)$ to $(3, -1)$    | $(0, -4)$ to $(2, -3)$   | $(-1, 3)$ to $(-1, 1)$   | $(3, -1)$ to $(3, -2)$   |
| $(-3, -3)$ to $(-3, -5)$ | $(-1, 0)$ to $(-1, -2)$  | $(2, 2)$ to $(4, 2)$     | $(0, -3)$ to $(0, -5)$   |
| $(-1, 0)$ to $(1, 0)$    | $(-4, -2)$ to $(-3, -2)$ | $(-1, 3)$ to $(1, 3)$    | $(-1, -1)$ to $(1, -1)$  |
| $(0, -4)$ to $(2, -5)$   | $(-1, 1)$ to $(1, 1)$    | $(-3, 0)$ to $(-3, -2)$  | $(2, 1)$ to $(4, 1)$     |
| $(4, 0)$ to $(3, -1)$    | $(-3, -3)$ to $(-1, -3)$ | $(-4, 3)$ to $(-2, 3)$   |                          |

What message is produced ?

## What have I learned ?

1. What temperature is shown here ?



2. I had £30 in my bank account and wrote a cheque for £50.

What was my new balance ?

3. Artimus was born in the middle of 26 B.C. and died at the end of 44 A.D.

How old was he when he died ?

4. What is the temperature that is :-

(a)  $7^{\circ}\text{C}$  up from  $-2^{\circ}\text{C}$  ?

(b)  $4^{\circ}\text{C}$  down from  $-11^{\circ}\text{C}$  ?

5. The temperature rose from  $-32^{\circ}\text{C}$  to  $-18^{\circ}\text{C}$ . By how much had it risen ?

6. Find the following :-

(a)  $3 + (-7)$

(b)  $(-2) + 8$

(c)  $(-10) + 4$

(d)  $(-6) + (-4)$

(e)  $(-7) + 7$

(f)  $6 - 7$

(g)  $2 - 11$

(h)  $(-5) - 3$

(i)  $(-18) - 12$

(j)  $4 + (-7)$

(k)  $0 + (-7)$

(l)  $(-6) + (-3)$

(m)  $(-7) + 5$

(n)  $6 \times (-2)$

(o)  $(-8) \times 4$

(p)  $0 \times (-5)$

(q)  $9 \times (-5)$

(r)  $(-12) \div 6$

(s)  $(-9) \div 9$

(t)  $(-50) \div 5$

7. Find the following :-

(a)  $(3 \times (-8)) \div 6$

(b)  $5 \times (-2) \times 4$

(c)  $[6 + (-3)] \times 5$

(d)  $(2 - 5) \times 7$

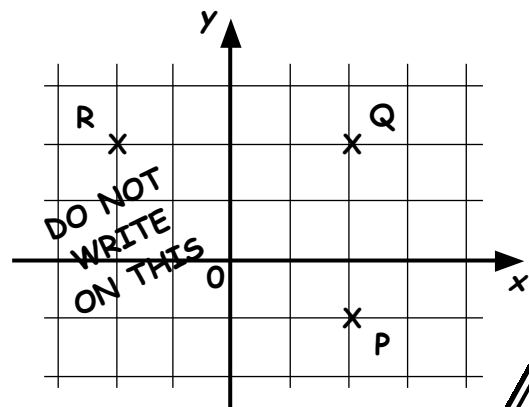
(e)  $-2 \times (-5 + 8)$

(f)  $(-40) + (-20) \div 10$

8. (a) Write down the coordinates of the three points, P, Q and R.

- (b) Find a fourth point, (call it S) such that PQRS is a rectangle.

Write down the coordinates of S.



9. (a) Draw a new set of axes, (-6 to 6 on each scale) and plot the points :-

A(-3, 1), B(4, 3), C(3, -1) and D(-4, -3).

- (b) Join  $A \rightarrow B \rightarrow C \rightarrow D$ . (c) What shape is ABCD ?