

Chapter 5

Negative Numbers

Negative Numbers

Definition An **INTEGER** is the more mathematical name for what you already know as a **negative** number.

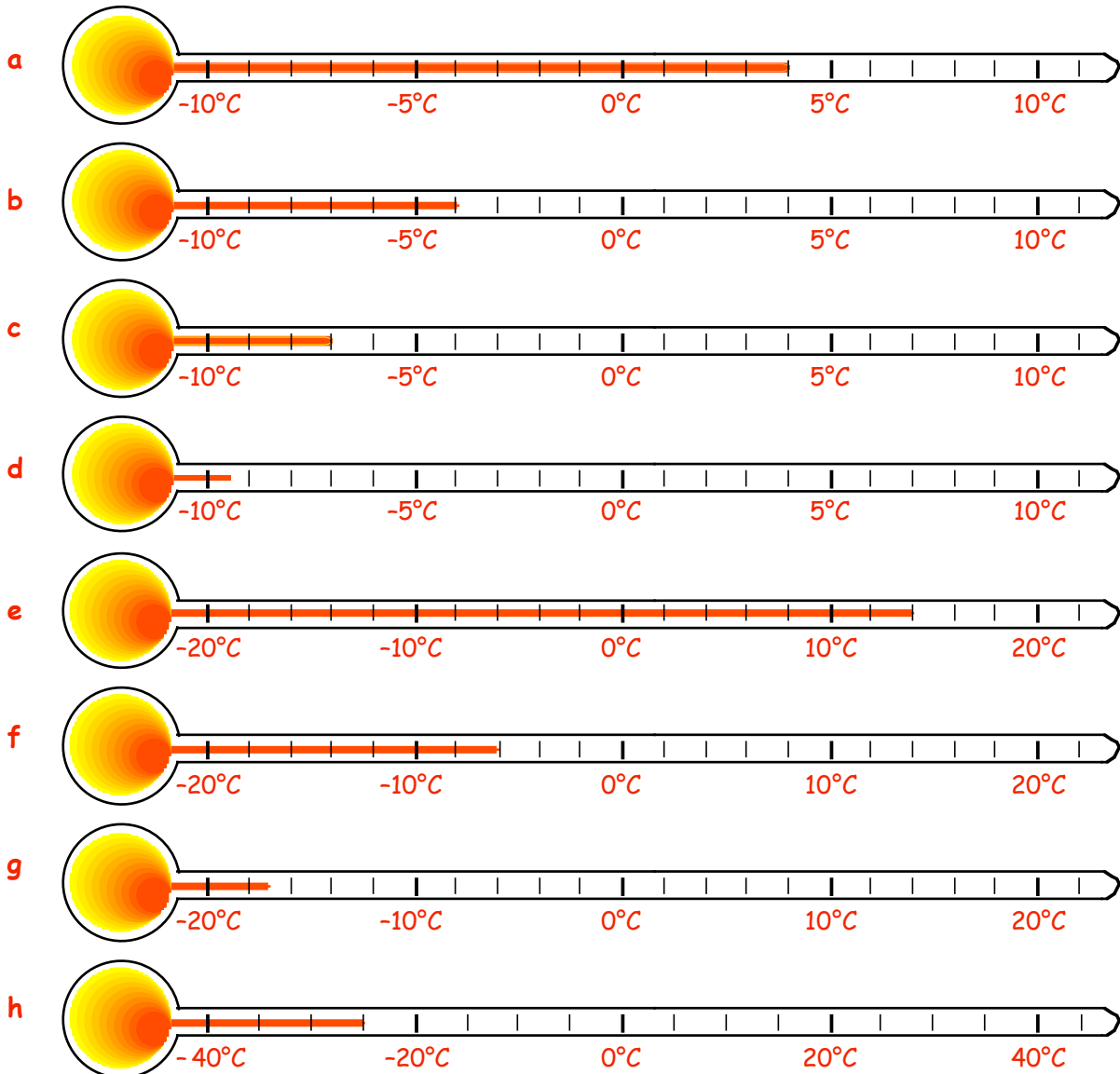
Strictly speaking, an **integer** is simply a **NEGATIVE** or **POSITIVE** whole number (including 0).

Examples :- -3, -29, 7, 31, 0, -1, 10000, -1903, etc. are all **INTEGERS**.
3.5, $\frac{3}{4}$, $2\frac{1}{2}$, -4.1, $1\frac{3}{4}$, -22.7, etc., are **NOT** integers. (Can you see why?)

Exercise 1

1. A **thermometer** is the most obvious place to see positive and negative numbers.

What temperatures are shown here :-



2. You will also come across negative numbers when dealing with money in a bank.

If a man has £65 in his bank account, the computer records this as

+£65.00



- a If he is "overdrawn" by £35, what do you think this will show up as ?
 b What do each of these "bank balances" really mean ?

(i)

<input type="radio"/>		<input type="radio"/>
<input type="radio"/>	15/01/09	<input type="radio"/>
<input type="radio"/>	balance + £18.80	<input type="radio"/>
<input type="radio"/>		<input type="radio"/>

(ii)

<input type="radio"/>		<input type="radio"/>
<input type="radio"/>	31/03/09	<input type="radio"/>
<input type="radio"/>	balance - £27.75	<input type="radio"/>
<input type="radio"/>		<input type="radio"/>

(iii)

<input type="radio"/>		<input type="radio"/>
<input type="radio"/>	06/06/09	<input type="radio"/>
<input type="radio"/>	balance - £125.00	<input type="radio"/>
<input type="radio"/>		<input type="radio"/>

(iv)

<input type="radio"/>		<input type="radio"/>
<input type="radio"/>	01/04/09	<input type="radio"/>
<input type="radio"/>	balance + £0.00	<input type="radio"/>
<input type="radio"/>		<input type="radio"/>

- c Dan had £35 in his bank account and he withdrew £40.
 What will the computer show his balance to be now ?

- d Diana's bank balance is shown opposite.
 She paid in £20 to her account.
 What will her new balance be ?

balance (- £45.00)

- e Richard's bank balance was £0.00. He withdrew £60.
 What will his new balance be ?

- f Last week Lucy's bank balance stood at (- £35.00).
 She withdrew a further £15.
 What will her balance be now ?



- g If Ted's bank balance stood at (- £57), how much must he
 deposit to "clear his overdraft" ?

- h Angela's balance showed + £23.50.
 She signed two cheques, one for £12.50 and another for £6.80.
 What will her new balance now show ?

- i My balance, at the end of last month, was (-£450).
 The next day, my salary of £1175 was paid in.
 What was my new bank balance ?

- j Nick's balance last week was (-£24). He signed a cheque
 for £35 and on the same day his pay of £380 was deposited
 in the bank.
 What was his new balance ?



Using Thermometers



A **thermometer** is a useful means of studying negative numbers.

Exercise 2 (No calculator)



- Use a ruler to copy this thermometer neatly into your jotter.
- Look at your thermometer.

What is the temperature that is :-

- | | | | |
|---|------------------------|---|-----------------------|
| a | 4°C up from 11°C ? | b | 6°C up from 0°C ? |
| c | 15°C up from 7°C ? | d | 9°C down from 15°C ? |
| e | 7°C down from 13°C ? | f | 5°C up from -3°C ? |
| g | 6°C down from -2°C ? | h | 15°C up from -5°C ? |
| i | 8°C down from 3°C ? | j | 22°C down from 0°C ? |
| k | 11°C down from -10°C ? | l | 18°C down from -3°C ? |
| m | 4°C up from -12°C ? | n | 25°C up from -30°C ? |

- Can you see that 8°C is "10°C up from" -2°C ?

Copy and complete these in the same way :-

(say whether it's " .. up from" or " .. down from" each time).

- | | | | |
|---|---------------------------|---|---------------------------|
| a | 10°C is ...°C up from 6°C | b | 3°C is from 10°C |
| c | 0°C is from 11°C | d | 5°C is from -2°C |
| e | -4°C is from 0°C | f | 3°C is from -8°C |
| g | -25°C is from -15°C | h | -6°C is from 6°C |
| i | 30°C is from -30°C | j | -45°C is from -30°C |

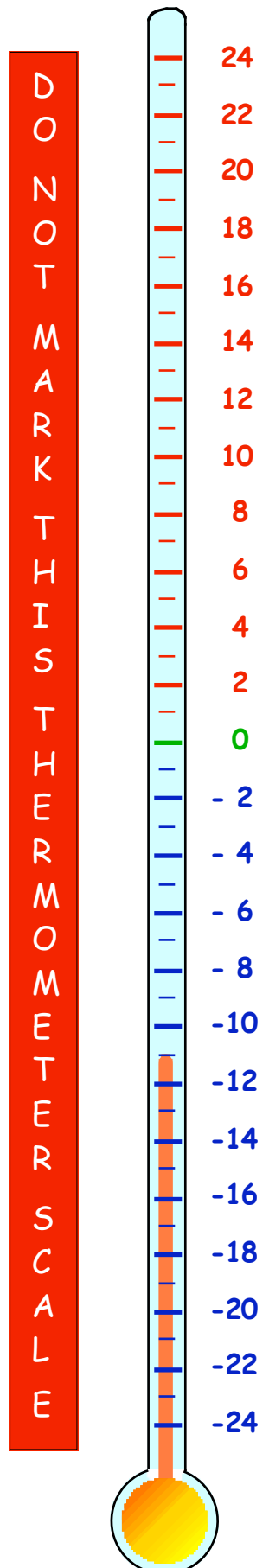
- One winter's day in Glasgow, the temperature was -7°C.
In Aberdeen it was 6° colder.

What was the temperature in Aberdeen ?



- When I left my hotel in Iceland, the temperature fell from 15°C to -17°C.

By how much had the temperature changed ?

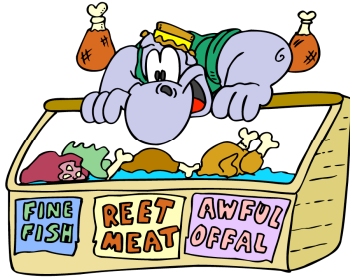


6. Whilst on holiday in Egypt, I noticed the temperature rose from -18°C at night to 32°C at noon in the desert.

By how much had the temperature **risen** ?



7.



When a butcher put a side of beef in his freezer, its temperature fell by a **steady amount** each hour.

It started at 11°C and fell to 7°C in one hour.

What would the temperature be after :-

- a 2 hours ? b 3 hours ?
c 4 hours ? d 7 hours ?

Adding and Subtracting Negatives

When adding and subtracting positive and negative numbers, the best way is to draw or imagine them as temperatures on a thermometer.

Example 1 :- To find $2 + 6$,
imagine the **2** on a thermometer.

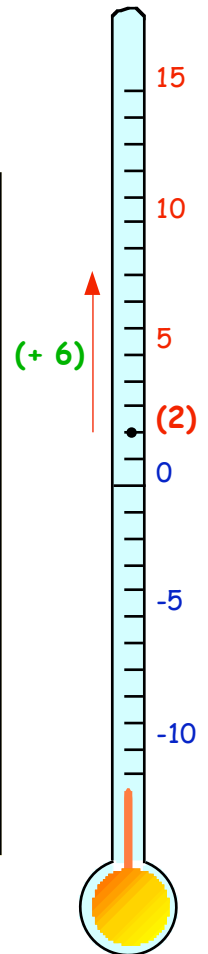
To do the "+6" bit, you go **UP** by 6 $\rightarrow 2 + 6 = 8$

Example 2 :- To find $2 + (-6)$,
imagine the **2** on a thermometer.

To do the "+(-6)" bit, you go **DOWN** by 6 $\rightarrow 2 + (-6) = -4$

Example 3 :- To find $7 - 10$,
imagine the **7** on a thermometer.

To do the "-10" bit \rightarrow you go **DOWN** by 10 $\rightarrow 7 - 10 = -3$



Exercise 3 (No calculator)

Use the **thermometer** which you drew from the last exercise, (or draw a new one), to help you here.



1. Write down each question first, then the answer :-

- | | | | | | | | |
|---|---------------|---|----------------|---|-------------|---|----------------|
| a | $4 + 9$ | b | $2 + 10$ | c | $0 + 5$ | d | $6 + (-4)$ |
| e | $7 + (-3)$ | f | $10 + (-10)$ | g | $6 + (-8)$ | h | $1 + (-7)$ |
| i | $0 + (-11)$ | j | $(-5) + 8$ | k | $(-9) + 9$ | l | $(-2) + 23$ |
| m | $(-12) + 5$ | n | $(-20) + 15$ | o | $4 + (-9)$ | p | $(-3) + (-12)$ |
| q | $(-6) + (-6)$ | r | $(-4) + (-16)$ | s | $(-14) + 5$ | t | $(-16) + 14$ |

2. Again use your thermometer to help here :-

(remember :- $4 - 6$ means "go to 4, then move **down** by 6").

a $7 - 5$

b $15 - 15$

c $7 - 1$

d $4 - 9$

e $3 - 12$

f $7 - 17$

g $0 - 11$

h $(-3) - 5$

i $(-8) - 6$

j $(-15) - 5$

k $(-1) - 19$

l $0 - 23$

m $29 - 49$

n $(-13) - 26$

o $200 - 500$

p $(-69) - 31$

3. A **Mixture** !! The rule is simple.

Picture the **first** number on your thermometer.

If you add a **positive** number move **UP**.

If you add a **negative** number or take away a number move **DOWN**.

a $3 + 8$

b $5 + (-9)$

c $1 - 8$

d $(-2) + 12$

e $-4 + (-6)$

f $13 - 15$

g $(-4) - 8$

h $(-30) + (-20)$

i $-20 + 35$

j $0 - 19$

k $0 + (-19)$

l $(-18) + (-3)$

m $18 + (-3)$

n $(-18) + 3$

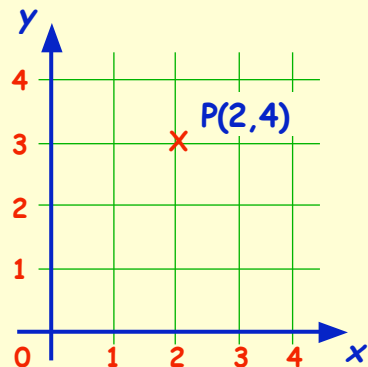
o $(-37) + 37$

p $54 - 86$

Coordinates

Revision :- You should know what a Coordinate diagram, (or a **Cartesian** diagram), looks like.

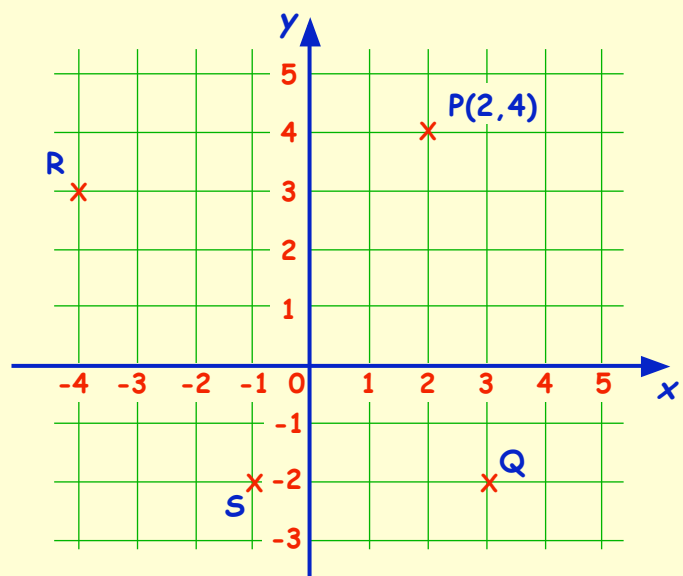
Remember :- x-axis (or horizontal axis).
y-axis (or vertical axis).
The origin (O).
P is 2 (**right**) and 4 (**up**) from the origin.
=> **P(2, 4)**, has x-coordinate 2 and y-coordinate 4.



We now extend the set of x and y axes **backwards** and **downwards**.

Look at the numbers on the x- and y- axes.

They now include **NEGATIVE** values.



Can you see, from the previous diagram, the following :-

the point Q is 3 (to the right) and 2 (down) from the origin → $Q(3, -2)$

the point R is 4 (to the left) and 3 (up) from the origin → $R(-4, 3)$

the point S is 1 (to the left) and 2 (down) from the origin → $S(-1, -2)$?

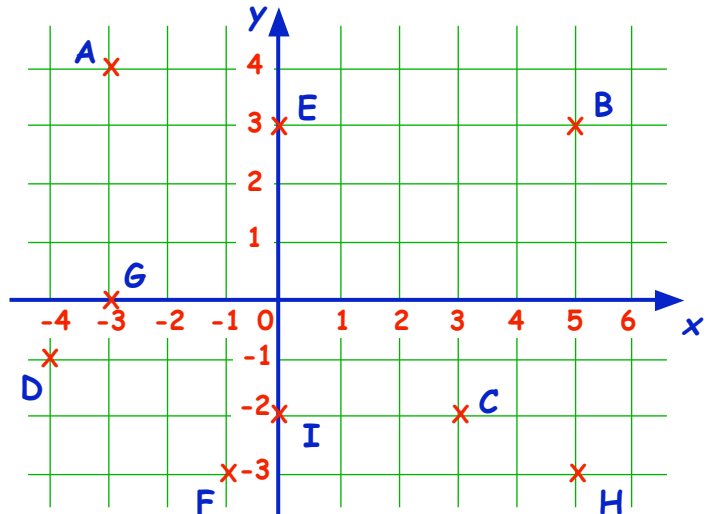
Exercise 4

1. Look at this coordinate diagram.

The coordinates of A are

$A(-3, 4)$

Write down the coordinates of the other 8 points.

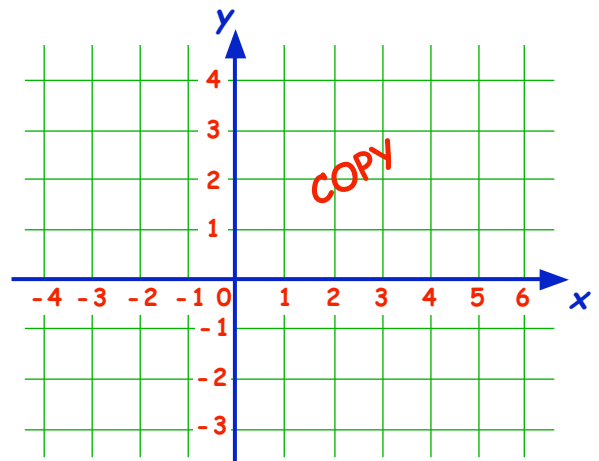


2. Draw a **large** set of axes (-10 to 10 on both scales).

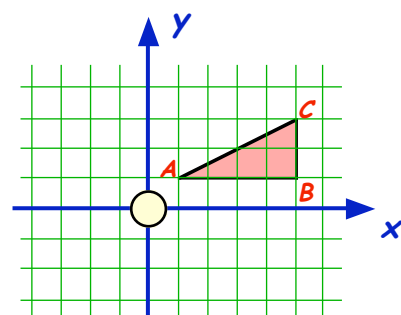
Plot each set of points, join them up and state what shape each is :-

- a $A(3,3)$ $B(5,4)$ $C(7,3)$ $D(5,-2)$. b $E(-7,5)$ $F(-5,8)$ $G(2,8)$ $H(0,5)$.
 c $I(-10,3)$ $J(-8,3)$ $K(-9,-3)$. d $L(1,-5)$ $M(-4,-4)$ $N(-5,1)$ $O(0,0)$.
 e $P(4,-5)$ $Q(6,-7)$ $R(5,-9)$ $S(3,-9)$ $T(2,-7)$.
 f $U(-8,-3)$ $V(-6,-3)$ $W(-5,-5)$ $X(-6,-7)$ $Y(-8,-7)$ $Z(-9,-5)$.

3. a Copy this diagram and plot the three points :-
 $P(-3,2)$, $Q(5,2)$ and $R(5,-3)$.
 b Try to find a 4th point, (call it S), such that PQRS is a **rectangle**. Show S on your diagram, and write down its coordinates.



4. Look at triangle ABC.
- a Write down the coordinates of the 3 points, A, B and C.
- b "Flip" $\triangle ABC$ over the x-axis. Write down the new coordinates of the corners of the triangle.
- c Now "flip" your new triangle left across the y-axis and write down the coordinates of the 3rd triangle.



5. a Draw a set of axes, (-6 to 6 on both scales) and plot the four points A(2,1), B(3,5), C(5,5), D(6,1).
- b Join the four points and state what type of shape is formed.
- c "Flip" each of the four points over the x-axis to form a new four-sided shape. (This is called "REFLECTING" the shape).
- d Write down the coordinates of the four corners of this new reflected shape.
6. Draw a new set of axes from -8 to 8 on both scales.
- a Plot the 4 points P(0,1), Q(-1,6), R(-4,7) and S(-5,2) and join them up.
- b Reflect your shape over the y-axis and write down the coordinates of your new shape.
- c Reflect the original shape over the x-axis and write down the coordinates of your new shape.

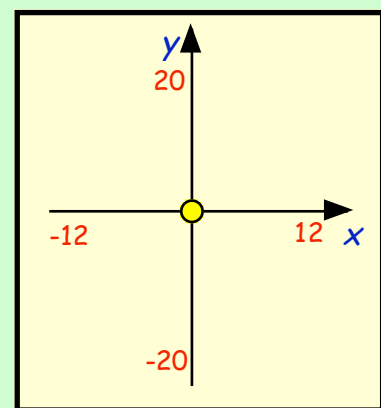
7. Take a new page in your jotter and in the **middle**, draw a set of axes **lightly** in pencil .

The x-axis is numbered -12 to 12.

The y-axis is numbered -20 to 20.

Use tiny dots to mark each point in the following sets and join them up neatly.

When all the sets, A to K, are drawn, a familiar figure should appear.



Page

Set A (-5,-16) (-3,-16) (-5,-11) (-3,-7) (9,-1) (10,2) (-3,-7) (-7,-7)
 (-10,-1) (-8,5) (-9,7) (-8,10) (-6,10) (-6,14) (-9,15) (-7,16)
 (-4,16) (-5,18) (5,14) (4,17)

Set B (6,17) (5,14) (9,14) (11,13) (9,12) (7,13) (8,14) (6,12) (4,11)
 (4,12) (2,14) (3,11) (-1,14) (-6,14)

Set C (6,12) (7,0)

Set D (4,9) (6,7) (4,5)

Set E (-5,-16) (-7,-14) (-5,-11) (-7,-7) (-9,-6) (-8,-5)

Set F (-4,-5) (-6,-1) (-4,-1) (1,2) (0,6) (2,1)

Set G (-8,5) (-6,4) (-3,5) (-1,8) (3,3) (2,-3)

Set H (-6,10) (-3,9)

Set I (-2,10) (-1,9) (1,6)

Set J (-4,13) (-5,11) (-4,10) (-3,12) (-4,13)

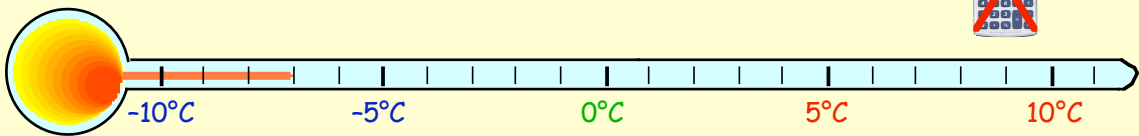
Set K (-1,13) (-2,11) (-1,10) (0,11) (-1,13). Have Fun !!!!! — **Who is it ?**

Topic in a Nutshell

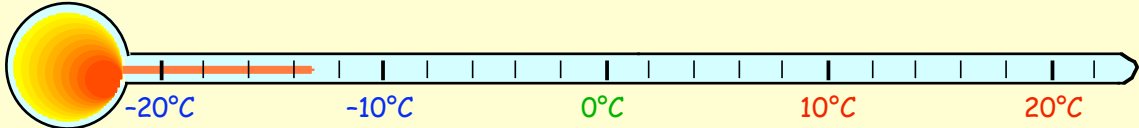
1. State what temperatures are represented on these thermometers :-



a



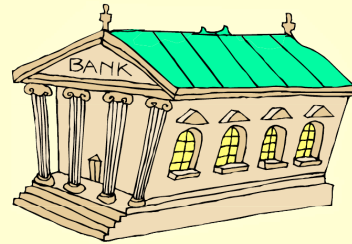
b



2. Jack's bank balance last month was (-£210).

This month his wage of £600 was paid into his account, but he also paid a phone bill of £145.

What is Jack's new bank balance ?



3. Write down what number is :-

a 4 up from -1

b 5 down from 3

c 8 down from -4

d 15 up from -9.

4. Find :-

a $5 + (-3)$

b $7 + (-7)$

c $(-4) + 9$

d $(-9) + 9$

e $(-9) + (-9)$

f $(-5) + (-17)$

g $(-12) + 11$

h $(-101) + 102.$

5. The temperature at midday in Gran Canaria was 24°C .

At midnight it had fallen to -3°C .

By how many degrees had the temperature fallen ?



6. Find :-

a $6 - 8$

b $5 - 12$

c $(-1) - 6$

d $(-4) - 9$

e $0 + (-5)$

f $(-6) - 1$

g $(-48) + 50$

h $23 - 52.$

7. a Copy this coordinate diagram.

b Plot the points :-

$A(-2, 1)$, $B(1, 3)$ and $C(3, 0)$.

c Find a 4th point, (call it D) so that figure ABCD is a **SQUARE**.

Show point D on your diagram and complete the drawing of the square.

